ABRHS PHYSICS (H) NAME: KEY

Momentum & Collisions

1. A 1500 kg car rear-ends a 2000 kg at a stop sign. Immediately after the collision, the two cars have a speed of 13 m/s. (They then skid to a stop, but that is not part of this problem.) How fast was the 1500 kg car moving just before the collision?

Mom. Conserved

$\therefore (1500)V + (2000)(0) = (1500)13 + (2000)(13)$ → $1500V = 45{,}500$

$\boxed{V = 30.3\ m/s}$

2. Maya and Miguel are on the bumper cars and have a head-on collision. Maya (total mass 300 kg) is traveling to the right at 5 m/s. Miguel (250 kg) is traveling to the left at 3 m/s. After the collision, Maya is only traveling at 1 m/s, but still to the right. What is the final velocity of Miguel?

$(300)(5) + (250)(-3) = (300)(1) + (250)V$ Mom. Conserved!

$1500 - 750 = 300 + 250V$

$450 = 250V$

$\boxed{V = 1.8\ m/s}$

3. A stream of 40 gram bullets, fired horizontally with a speed of 1000 meters per second, strikes a 10 kg wooden block that is free to move on a horizontal frictionless tabletop. What is the speed of the block after it has absorbed 15 bullets?

$(15)(.04)(1000) = (10)V + (15)(.04)V$

$600 = (10.6)V$

$\boxed{V = 56.6\ m/s}$

4. A 0.5 kg ball is traveling with a speed of 7 m/s when it collides with a 0.4 kg ball traveling in the opposite direction with a speed of 3 m/s. After the collision, the first ball (0.5 kg) is traveling with a speed of 1 m/s, in the same direction that it was before.
   a. What is the speed and direction of the second ball after the collision?

   $(0.5)(7) + (0.4)(-3) = (0.5)(1) + (0.4)V$ → $1.8 = .4V$

   $3.5 - 1.2 = .5 + .4V$

   $\boxed{V = 4.5\ m/s}$

   b. Which ball experienced the greater change in momentum? change in velocity? impulse?

   Δp = same! J = same! ΔV → smaller mass = greater ΔV

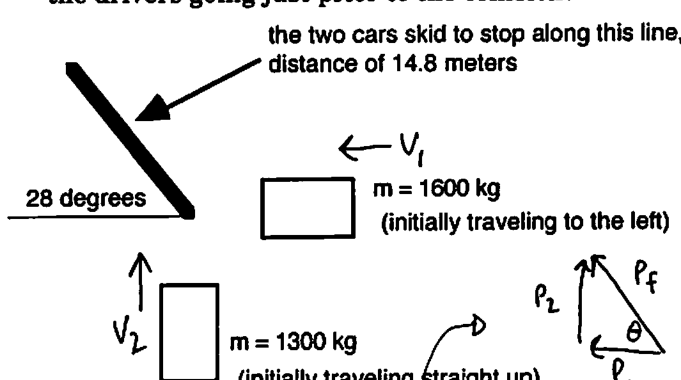
   c. If the collision lasted for 0.25 seconds, what was the force on the second ball?

   $J = Ft$ & $J = \Delta p$

   $\therefore Ft = \Delta p$ → $F(.25) = (.4)(4.5) - (.4)(-3)$

   $F(.25) = 1.8 + 1.2 = 3$ → $\boxed{F = 12\ N}$

5. Two cars approach a 90° intersection. Neither driver is paying attention to what they are doing, and they collide. After the collision, the cars stick together, and skid to a stop in 14.8 meters at an angle as shown. The two cars have masses of 1300 kg and 1600 kg, as shown. You happen to know that the coefficient of friction between the tires and the road was $\mu = 0.3$. How fast were the drivers going just prior to the collision?



From skidding! car loses all its K to friction

so $K - W = 0$

$\therefore \frac{1}{2}mv^2 = fd = \mu mgd$

$\therefore V^2 = 2\mu gd = 2(.3)(10)(14.8)$

$V^2 = 88.8$

$\underline{V = 9.42\ m/s}$ ← This is speed right after collision.

Momentum conserved! Vector!

$\vec{P}_{1_i} + \vec{P}_{2_i} = \vec{P}_f$

$\therefore (1600)V_1 = (1300+1600)(9.42)\cos 28$ → $\boxed{V_1 = 15\ m/s}$

$(1300)V_2 = (1300+1600)(9.42)\sin 28$ → $\boxed{V_2 = 9.9\ m/s}$

side 1

6) Elastic, so Mom & Kinetic conserved → Hey! 2 Equations, 2 unknowns.

$$\vec{p}: \quad m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f}$$

$$K: \quad \tfrac{1}{2} m_1 v_{1_i}^2 + \tfrac{1}{2} m_2 v_{2_i}^2 = \tfrac{1}{2} m_1 v_{1_f}^2 + \tfrac{1}{2} m_2 v_{2_f}^2$$

Rewrite: $\vec{p} \Rightarrow \quad m_1 v_{1_i} - m_1 v_{1_f} = m_2 v_{2_f} - m_2 v_{2_i}$

$K \Rightarrow \quad m_1 v_{1_i}^2 - m_1 v_{1_f}^2 = m_2 v_{2_f}^2 - m_2 v_{2_i}^2$

divide & masses cancel!

divide them to get:

$$v_{1_i} + v_{1_f} = v_{2_f} + v_{2_i}$$

$$\hookrightarrow \underline{\underline{v_{1_i} = v_{2_f} + v_{2_i} - v_{1_f}}}$$

Sub into momentum:

$$m_1 (v_{2_f} + v_{2_i} - v_{1_f}) + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f}$$

$$m_1 v_{2_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f} - m_1 v_{2_f} + m_1 v_{1_f}$$

$$(.25 + .35)\, v_{2_i} = (.25)(6.5) + (.35)(9) - (.25)(9) + (.25)(6.5)$$

$$.6\, v_{2_i} = 3.25 + 3.15 - 2.25$$

$$.6\, v_{2_i} = 4.15$$

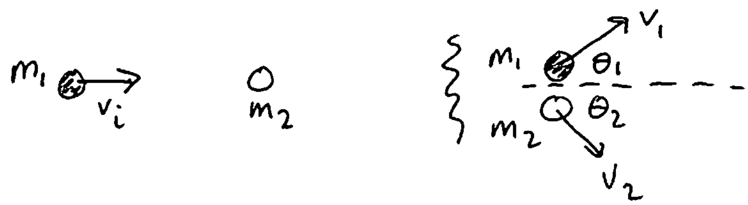
$$\boxed{v_{2_i} = 6.92 \text{ m/s}}$$

& $\quad v_{1_i} = v_{2_f} + v_{2_i} - v_{1_f} = 9 + 6.92 - 6.5$

$$\boxed{v_{1_i} = 9.42 \text{ m/s}}$$

[or use blobby equations]

7) 2-D collision. Momentum is conserved → vector!



So $\boxed{m_1\vec{v}_i = m_1\vec{v}_1 + m_2\vec{v}_2}$ vector equation. use components or geometry.

I) using components:

$$m_1 v_i\,\hat{\imath} + 0\hat{\jmath} = (m_1 v_1 \cos\theta_1\,\hat{\imath} + m_1 v_1 \sin\theta_1\,\hat{\jmath}) + (m_2 v_2 \cos\theta_2\,\hat{\imath} - m_2 v_2 \sin\theta_2\,\hat{\jmath})$$

$\therefore$ $\hat{\imath}$) $m_1 v_i = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$

$\hat{\jmath}$) $0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2$

$m_1 = 0.2$ kg $\quad m_2 = 0.4$ kg
$v_i = 3.5$ m/s $\quad v_2 = ?$
$v_1 = 3$ m/s $\quad \theta_2 = ?$
$\theta_1 = 30°$

so $(.2)(3.5) = (.2)(3)(\cos 30) + (.4)v_2 \cos\theta_2$

$0 = (.2)(3)(\sin 30) - (.4)v_2 \sin\theta_2$

so $0.7 = 0.52 + .4\, v_2 \cos\theta_2$

$\underline{0.451 = v_2 \cos\theta_2}$] → That is v_x!

And $0 = 0.3 - .4\, v_2 \sin\theta_2$

$\underline{.75 = v_2 \sin\theta_2}$] → That is v_y

$\therefore$ $\boxed{\vec{v}_2 = 0.45\,\hat{\imath} - .75\,\hat{\jmath} \text{ m/s}}$

$$\therefore \frac{v_2 \sin\theta_2}{v_2 \cos\theta_2} = \frac{.75}{.451} \Rightarrow \tan\theta_2 = 1.66$$

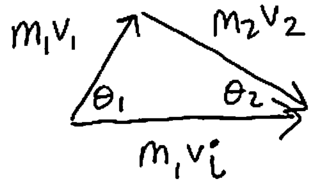
$\boxed{\theta_2 = 59°}$

so $0.451 = v_2 \cos(59)$ $\Rightarrow$ $\boxed{v_2 = .875 \text{ m/s}}$

↳ so $\vec{v}_2 = 0.875$ m/s @ 59° (below)

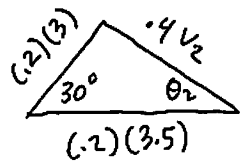
7) done as diagram

$$m_1\vec{v}_i = m_1\vec{v}_1 + m_2\vec{v}_2$$

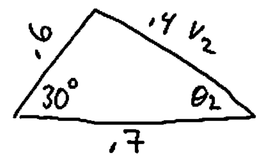


a triangle! (not nec. right triangle.)

So



$\Rightarrow$



$$(.4v_2)^2 = (.6)^2 + (.7)^2 - 2(.6)(.7)\cos 30$$ (Law of Cosines)

$$.16v_2^2 = .36 + .49 - .727$$

$$v_2^2 = 0.766$$

$$\boxed{v_2 = .875 \text{ m/s}}$$

now $.6\sin 30 = (.4)(.875)\sin\theta_2$

$$\sin\theta_2 = .857$$

$$\boxed{\theta_2 = 59°}$$

i.e. $\vec{v}_2 = 0.875$ m/s @ $59°$ below horizontal

8) Either with components or a picture:

a)
$m_1 = 0.25$ kg $\quad m_2 = 0.15$ kg

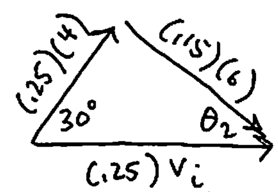
$v_i = ?$ $\quad v_2 = 6$ m/s

$v_1 = 4$ m/s $\quad \theta_2 = ?$

$\theta_1 = 30°$

$$(.25)v_i\,\hat{i} = \left((.25)(4)\cos 30\,\hat{i} + (.25)(4)\sin 30\,\hat{j}\right) + (.15)(6)\cos\theta_2\,\hat{i} - (.15)(6)\sin\theta_2\,\hat{j}$$

or



$$(.15)(6)\sin\theta_2 = (.25)(4)\sin 30$$

$$\sin\theta_2 = 0.556$$

$$\boxed{\theta_2 = 33.7°}$$

$$(.25)v_i = (.25)(4)\cos 30 + (.15)(6)\cos 33.7$$

$$\boxed{v_i = 6.46 \text{ m/s}}$$

So $K_i \stackrel{?}{=} K_f$ $\qquad K_i = \frac{1}{2}(.25)(6.46)^2 = \underline{\underline{5.21 \text{ J}}}$

$$K_f = \frac{1}{2}(.25)(4)^2 + \frac{1}{2}(.15)(6)^2 = \underline{\underline{4.7 \text{ J}}}$$

Lost kinetic energy, so not elastic.

b) $J = \Delta p$! Since $\Delta\vec{p}_1 = -\Delta\vec{p}_2$ $\quad$ & $\quad \Delta\vec{p}_2 = \vec{p}_{2_f} - 0$

$\Delta\vec{p}_2 = (.15)(6) = 0.9$ kg·m/s @ 33.7° below horizontal

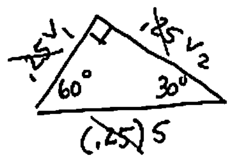
$\therefore -\Delta p_2 = 180°$ away (opposite direction)

$$\boxed{\text{So } \Delta p_1 = 0.9 \text{ kg·m/s @ } 146.3°}$$

(or in $\hat{i}\,\hat{j}$ $\quad -.75\,\hat{i} + 0.50\,\hat{j}$ N·s)

9) $m_1 = m_2$ & Elastic. $\therefore \vec{v}_1 \perp \vec{v}_2$!

So momentum looks like:



(m's cancel)

So $v_1 = 5 \cos 60$

$$\boxed{v_1 = 2.5 \text{ m/s}}$$

and $v_2 = 5 \cos 30$

$$\boxed{v_2 = 4.33 \text{ m/s}}$$