NAME: _____

Center of Mass

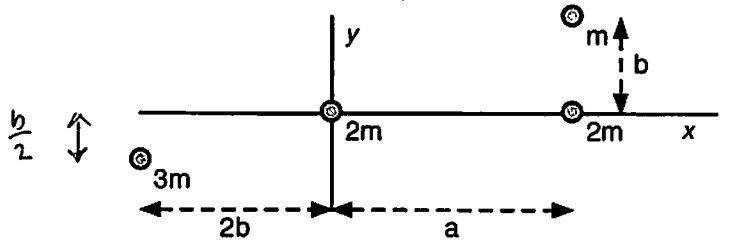
$$r = \frac{1}{M}\int x\,dm$$

In three dimensions we would just treat each dimension independently - so we would have three different summations (or integrals), one for each dimension.

Why is the center of mass important? Basically, when we do Newton's Laws involving large objects, the Second Law applies to the center of mass. Add up all the external forces acting on an object as if they were acting on the center of mass. This is what we did earlier in the year - and it will still apply in the future when we look at rotation.

Questions:

1. Four point masses are arranged as shown in the diagram. Notice that the origin passes through one of the masses labeled "2m." Where is the center of mass? (Worded another way, what is the center of mass vector?)



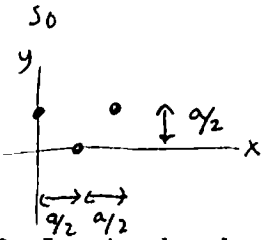
$$y = \frac{\sum m_i y_i}{M} = \frac{(3m)(-\frac{b}{2}) + (2m)(0) + (2m)(0) + (m)(b)}{3m + 2m + 2m + m} = \frac{-\frac{3}{2}mb + mb}{8m} = \frac{-\frac{1}{2}b}{8} = -\frac{1}{16}b$$

$$x = \frac{\sum m_i x_i}{M} = \frac{(3m)(-2b) + (2m)(0) + (2m)(a) + (m)(a)}{3m + 2m + 2m + m} = \frac{-6mb + 2ma + ma}{8m} = \frac{-6b + 3a}{8}$$

So $\vec{r} = \left(\frac{-6b+3a}{8}\hat{i} - \frac{b}{16}\hat{j}\right)$

2. Three identical, uniform masses each of mass m and length a are arranged as three sides of a square as shown in the diagram. Where is the center of mass?

The center of mass of ℓ segment is the center of ℓ segment



So $x = \frac{\sum m_i x_i}{3m} = \frac{1}{3m}\left(0 + \frac{ma}{2} + ma\right) = \frac{1}{3m}\left(\frac{3}{2}am\right) = \frac{1}{2}a$

$y = \frac{\sum m_i y_i}{3m} = \frac{1}{3m}\left(\frac{ma}{2} + 0 + \frac{ma}{2}\right) = \frac{1}{3m}(ma) = \frac{1}{3}a$

$\vec{r} = \frac{1}{2}a\hat{i} + \frac{1}{3}a\hat{j}$

3. Imagine that there are a bunch ~~of different masses arranged~~ on the x-axis, on either side of the origin. After doing $r = \frac{\sum m_i x_i}{M}$ for all the masses, you get r = 0. What does that mean?

That means the origin is the center of mass!