1-D Elastic

① $m_1 = m_2$, $v_{2i} = 0$

So using blobs:

$$v_{2f} = \frac{2m}{2m} v_{1i} + \frac{0}{2m} 0$$

$$\boxed{v_{2f} = v_{1i}}$$

$$v_{1f} = \frac{0}{2m} v_{1i} + \frac{2m}{2m} 0$$

$$\boxed{v_{1f} = 0}$$

② Just $m_1 = m_2$

$$v_{2f} = \frac{2m}{2m} v_{1i} + \frac{0}{2m} v_{2i}$$

$$v_{1f} = \frac{0}{2m} v_{1i} + \frac{2m}{2m} v_{2i}$$

$$\boxed{\begin{aligned} v_{2f} &= v_{1i} \\ v_{1f} &= v_{2i} \end{aligned}}$$

③ $v_{2i} = 0$, $m_2 \gg m_1$

$$v_{2f} = \frac{2m_1}{m_1+m_2} v_{1i} + (\,)0$$

↑ limits. if $m_2 \gg m_1$ → imagine $m_2 \to \infty$ or $m_1 \to 0$

$$\frac{2m_1}{m_1+m_2} \approx 0$$

$$\boxed{\therefore v_{2f} \approx 0}$$ (nothing happens to big object)

$$v_{1f} = \frac{m_1-m_2}{m_1+m_2} v_{1i} + (\,)0$$

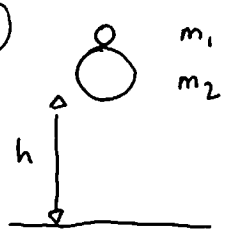
$$v_{1f} \approx (-1) v_{1i} \Rightarrow \boxed{v_{1f} = -v_{1i}}$$ little mass reverses itself.

④ $v_{2i} = 0$, $m_2 \ll m_1$

Same thing → but different limits, so

$$\frac{2m_1}{m_1+m_2} \approx 2 \quad \text{and} \quad \frac{m_1-m_2}{m_1+m_2} \approx 1$$

$$\boxed{\text{So } v_{2f} = 2v_{1i} \text{ & } v_{1f} = v_{1i}}$$

(5) 

They both hit "ground" @ same speed

↳ $U = K \rightarrow mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

Think of the collision as two separate collisions. First, m_2 crashes with earth, then m_2 crashes into m_1

First. Since $m_2 \ll M_{earth}$, it's velocity after the collision with the earth will be reversed, so $\sqrt{2gh}$ up.

Then. m_2 has head-on elastic collision with a much smaller m_1, they have the same speed, but opposite velocities.

∴ $m_2 \gg m_1$ and $v_{2i} = -v_{1i}$ Find v_{1f}

So $v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$

$v_{1f} \approx -v_{1i} + 2(-v_{1i})$

$v_{1f} \approx -3v_{1i}$

So little mass is now going up with 3x the speed = 9x kinetic energy

So it goes 9x higher → 9h

(6) ~~Again~~ $v_{2i} = -v_{1i}$ & $v_{2f} = 0$

So $0 = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}(-v_{1i})$

$2m_1 = m_2 - m_1$ $\quad m_2 = 3m_1$

∴ $v_{1f} = \frac{m_1 - 3m_1}{m_1 + 3m_1} v_{1i} + \frac{2(3m_1)}{m_1 + 3m_1}(-v_{1i}) = -\frac{1}{2}v_{1i} - \frac{3}{2}v_{1i} = -2v_{1i}$

$v_{1f} = -2v_{1i}$ → it bounces up w/ twice the speed so four times the kinetic energy so Four times as high

4h