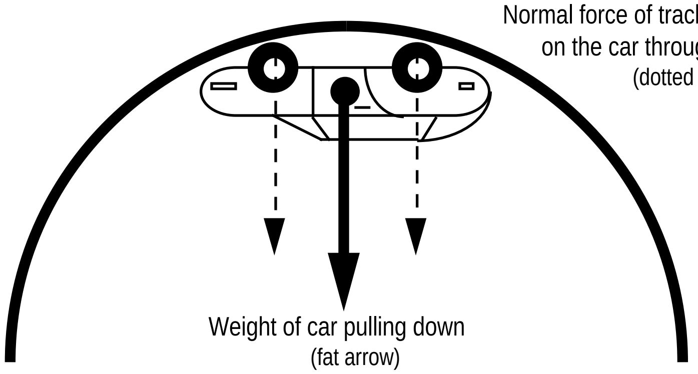
Lab 8-3: Supplement

Derivation of formula to find minimum height.
We need to find the minimum height from which to release the Hot Wheels car so that it just barely completes a loop-the-loop. We will assume that there is no friction in this derivation.

While the car is in the loop-the-loop, it is traveling in a circle. The net force on the car must be directed towards the center of that circle at all times; there is a centripetal force acting on the car! Looking at the very top of the loop-the-loop, while the car is upside down, the centripetal force comes from two individual forces: the weight of the car and the Normal force of the track pushing on the car. (*see diagram below*)



The normal force and the weight together act as a centripetal force to keep the car moving in a circle. The weight of the car is constant. The normal force, however, depends on how fast the car is traveling. If the car is going very fast, the normal force will be very large. The slower the car is moving, the less the normal is. The smallest value the normal force can have is zero. This means the track is no longer pushing on the car, and the only force acting on the car is the car's own weight. *In order for the car to just make the loop-the-loop, the normal force must be zero, and the weight of the car is the centripetal force making the car travel in a circle.* In equation form:

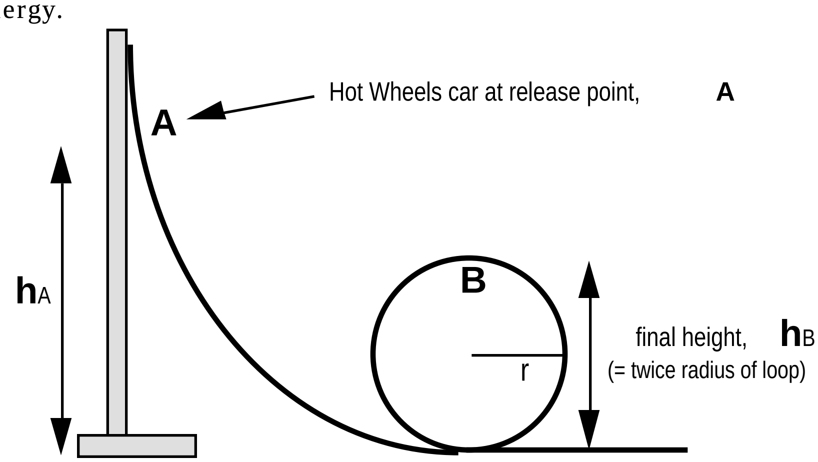
$$weight = centripetal\_force$$

$$mg = \frac{mv^2}{r}$$

After doing a little algebra, this equation becomes

$$v^2 = gr \quad \text{(equation 1)}$$

Equation 1 tells us how fast the car must be going in order to just stay on the track of a given radius, r, but it does not tell us how high the car must be released on the Hot Wheels track. We need another equation that relates speed and height; let's look at kinetic and potential energy.



Lab 8-3: Supplement

By placing the car at a certain height, we are giving it gravitational potential energy. When we release the car, its potential energy is transformed to kinetic energy. If we assume that there is no friction, the kinetic energy of the car at any point on the track is just equal to the change in its potential energy (because it is at a lower point on the track.) Because of the conservation of energy, *the potential energy the car has at the top of the Hot Wheels track is equal to the sum of it kinetic energy plus its lower potential energy at the top of the loop-the-loop*. In equation form:

$$\text{Potential Energy}_A = \text{Potential Energy}_B + \text{Kinetic Energy}_B$$

$$PE_A = PE_B + KE_B$$

$$mgh_A = mgh_B + \tfrac{1}{2}mv^2{}_B$$

Now we can substitute in equation 1, and simplify the energy equation.

$$mgh_A = mgh_B + \tfrac{1}{2}m(gr)$$

$$h_A = h_B + \tfrac{1}{2}r$$

We are almost done! The height of the car at the top of the loop-the -loop, h_B, is just twice the radius of the loop (h_B=2r), so we can rewrite the equation as

$$h_A = (2r) + \tfrac{1}{2}r$$

so the minimum height, h_A, is simply

$$\boxed{h_A = 2.5r}$$

So the minimum height needed for the car to just make the loop is two and a half times the radius of the loop!