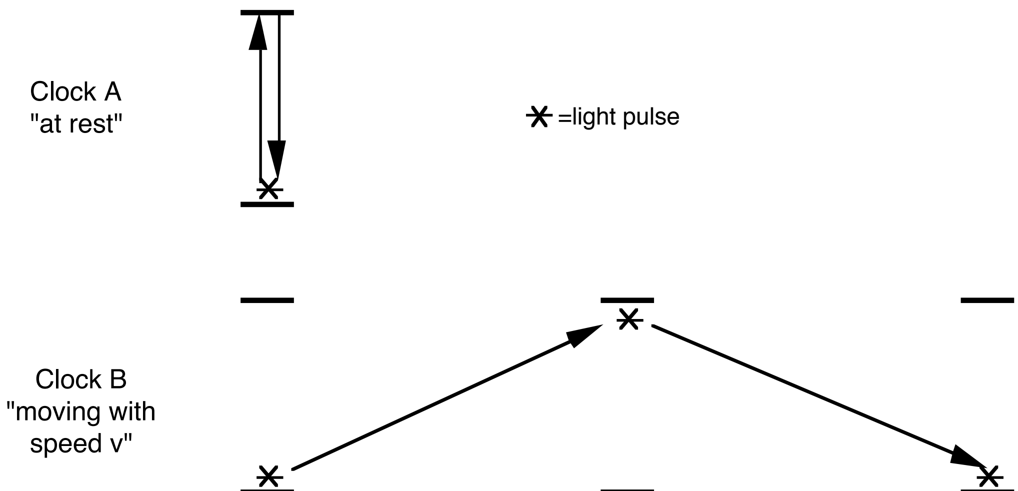
Time Dilation

Maxwell's equations show that an electromagnetic wave can only move at one speed, which depends on two fundamental constants. Since light is an electromagnetic wave, light can travel at only one speed, $c = 3 \times 10^8$ m/s. Special relativity comes from the simple idea that the laws of physics are the same for every one, no matter the reference frame. Surprisingly, this means that our concepts of time and space are not as rigidly defined as we usually think, and that different observers see time and space differently. Let's show the idea of time dilation first.

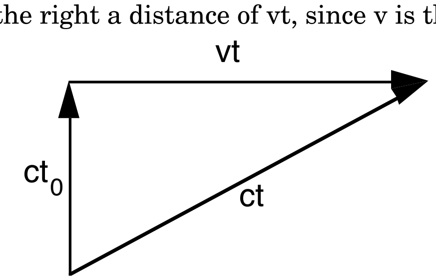
To show time dilation, we will follow the classic method and invent a clock that depends on light pulses. The clock consists of two mirrors, with a light pulse, or photon, that travels between the mirrors. The photon would happily bounce between the two mirrors at a constant rate, much like the swinging of a pendulum, and thus keep time for us.

Now imagine there are two such light clocks are moving with respect to each other. From Clock A's frame of reference, Clock A is at rest and Clock B is moving to the right with speed v. (Of course, B would see the exact opposite, but we'll come back to that later.)



While a person sitting next to Clock A would see Clock A behaving normally, Clock B would be odd. From A's frame of reference, the light pulse would have to travel a much longer distance to get to the second mirror and back again. Since the photon has to travel at a constant speed, A would think that the photon took a much longer to make the round trip. A would think that B's clock was running slow! A little geometry shows how much slower.

Assume it takes a time of t_0 for the photon to travel from one mirror to the next for Clock A. From A's perspective, it takes a time of t for the photon in Clock B to travel from one mirror to the next. The distances each photon traveled, from A's frame of reference, are then ct_0 and ct, while A sees B travel to the right a distance of vt, since v is the speed of B.



It is easy to then make the right triangle which shows this situation, and apply the Pythagorean Theorem, so that

$$(ct)^2 = (ct_0)^2 + (vt)^2$$

Time Dilation

Solving for t gives us

$$\left(c^2 - v^2\right)t^2 = (ct_0)^2$$

$$t^2 = \frac{c^2 t_0^2}{c^2 - v^2} = \frac{t_0^2}{1 - v^2/c^2}$$

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

This is sometimes written as $t = \gamma t_0$, where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Since γ is always greater than 1 for non-zero speeds v, A will always see B's clock taking more time between "photon ticks" than its own clock. However, as can be seen in the chart, B must be moving very close to the speed of light for this effect to be large.

V	γ
8000 m/s (space shuttle)	1.00
0.1 c	1.01
0.5 c	1.15
0.9 c	2.29
0.99 c	7.09

In the chart, c is the speed of light and is 3×10^8 m/s, so the speed of the space shuttle in orbit around the earth is about 8000 m/s, which is about 0.000027 c.

What must be remembered is that we could have done this derivation from B's frame of reference, in which case B would think it was normal and that A's clock was the slow one. All of the relativistic effects are relative. An observer will always perceive their own time to be "normal" and the time of someone moving past them as slow. A thinks B is slow, and B thinks A is slow, and they both are correct.