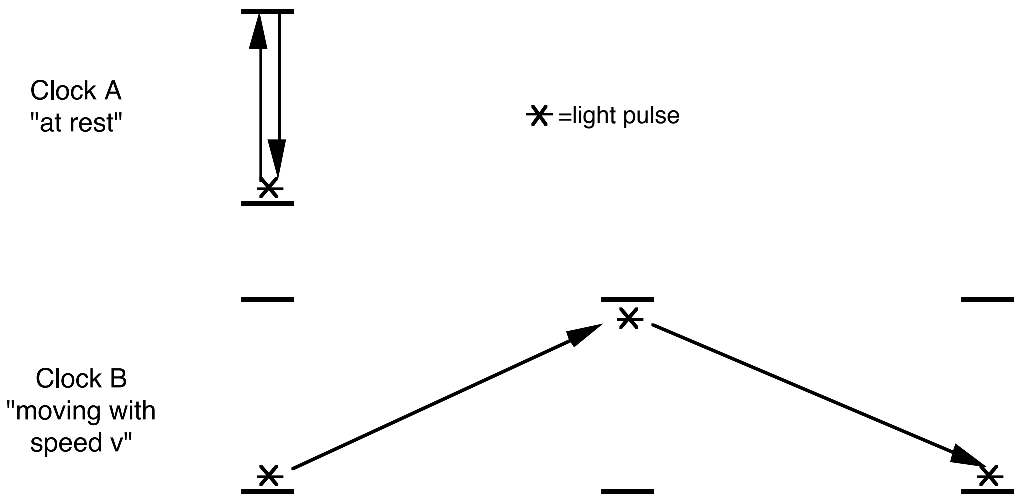
Time Dilation

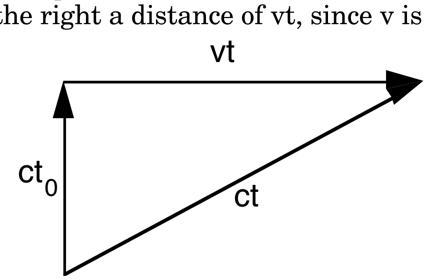
To show time dilation, we will follow the classic method and invent a clock that depends on light pulses. The clock consists of two mirrors, with a light pulse, or photon, that travels between the mirrors. The photon would happily bounce between the two mirrors at a constant rate, much like the swinging of a pendulum, and thus keep time for us.

Now imagine there are two such light clocks are moving with respect to each other. From Clock A's frame of reference, Clock A is at rest and Clock B is moving to the right with speed v. (Of course, B would see the exact opposite, but we'll come back to that later.)



While a person sitting next to Clock A would see Clock A behaving normally, Clock B would be odd. From A's frame of reference, the light pulse would have to travel a much longer distance to get to the second mirror and back again. Since the photon has to travel at a constant speed, A would think that the photon took a much longer to make the round trip. A would think that B's clock was running slow! A little geometry shows how much slower.

Assume it takes a time of t_0 for the photon to travel from one mirror to the next for Clock A. From A's perspective, it takes a time of t for the photon in Clock B to travel from one mirror to the next. The distances each photon traveled, from A's frame of reference, are then ct_0 and ct, while A sees B travel to the right a distance of vt, since v is the speed of B.



It is easy to then make the right triangle which shows this situation, and apply the Pythagorean Theorem, so that

$$(ct)^2 = (ct_0)^2 + (vt)^2$$

Solving for t gives us

$$\left(c^2 - v^2\right)t^2 = (ct_0)^2$$

$$t^2 = \frac{c^2 t_0^2}{c^2 - v^2} = \frac{t_0^2}{1 - v^2/c^2} \rightarrow t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

Time Dilation

Using the Lorentz factor, the equation for time dilation becomes

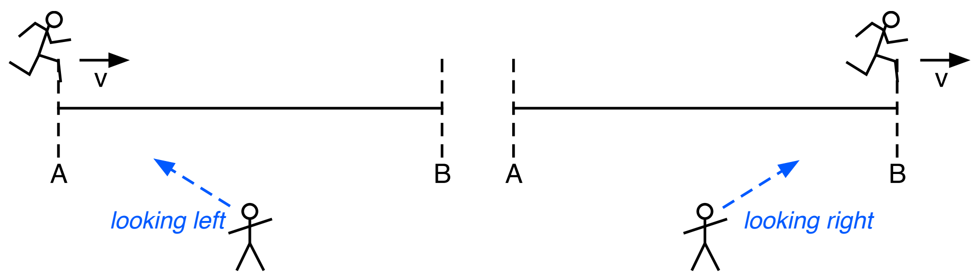
Time Dilation:	$t = \gamma t_0$

What must be remembered is that we could have done this derivation from B's frame of reference, in which case B would think it was normal and that A's clock was the slow one. All of the relativistic effects are relative. An observer will always perceive their own time to be "normal" and the time of someone moving past them as slow. A thinks B is slow, and B thinks A is slow, and they both are correct.

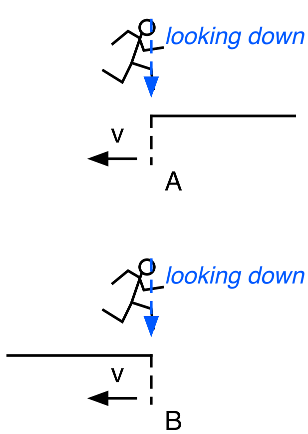
Proper Time

In the equation for time dilation, $t = \gamma t_0$, there are two different times. "t" is the time dilated time and "t_0" is the proper time. Both of those symbols are measuring the time between two events. (Think of starting a stopwatch and then stopping a stopwatch.) If the two events happen at the same location, that is the proper time, otherwise it is a time dilated time. There is only one reference frame that measures the proper time between two events - all others measure a time-dilated time, because there is only one reference frame in which the two events happen at the same coordinate. Let's do an example to help make that clearer.

Imagine someone timing a person running to the right with a speed v from A to B, shown below.



They would look at point A, and start timing when the runner crossed the line. Then they would look at point B, which is at a different location, and stop timing when the runner crosses that line. The person doing the timing would have measured a "t" - the time dilated time because the two events occurred at different coordinates in their reference frame.



But from the runner's point of view, shown to the left, they are just looking at their feet, and the lines pass by them to the left at speed v. They would start timing when the line for A passes under their feet and then stop when the line for B passes under their feet. They would have measured a "t_0" - the proper time because the two events happened at the same coordinate in their reference frame. It turns out, the person watching and timing would measure a longer time than the person running! The effect is unnoticeable at running speeds - but if they were moving close to the speed of light it is very noticeable.