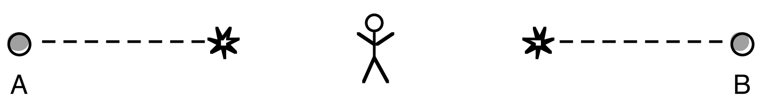
Simultaneity

Introduction
Special relativity forces us to really think about many things we used to take for granted. It also forces us to make sure that we carefully define many concepts that we don't ordinarily think too much about in our small, slow-moving world. We will imagine a series of "thought-experiments" to guide our thinking. These are experiments which may not be practical to do, but allow us to think deeply about the physics of situations involving very high speeds. We will also see that "common-sense" means very little (unless you happen to have learned special relativity already.)

Like many of the relativistic thought experiments, we start with a seemingly simple question: what does it mean for two events to happen at the same time if they are in different places? To answer this, we need to define what we mean by *reference frame*. In terms of special relativity, a reference frame is defined by all the points of view that do not have a relative velocity, meaning they are not moving with respect to each other. It doesn't matter where you are in your reference frame – two people who are not moving with respect to each other, even if separated by huge distances, are in the same reference frame. If two people are moving with respect to each other, even if right next to each other, are in different reference frames. (Strictly speaking, special relativity only deals with *inertial* reference frames, which are reference frames that only move with constant velocity with respect to each other. General relativity deals with gravity and non-inertial reference frames that are accelerating.)

Defining Reference Frames
We will begin by saying that anyone in a particular reference frame must agree on the order that events happen and whether two events happen at the same time or not. We will start off looking at one position in a reference frame, and then generalizing.



observer at midpoint of lights

Imagine that there are two flashes of light, and we have to decide if they flashed at the same time. If we stand right in middle of the two lights, as in the above diagram, it is easy to figure out. Because they are the same distance away from us, if they flash at the same time, then the light will reach us at the same time, so we would directly observe them flashing at the same time.

observer closer to light A

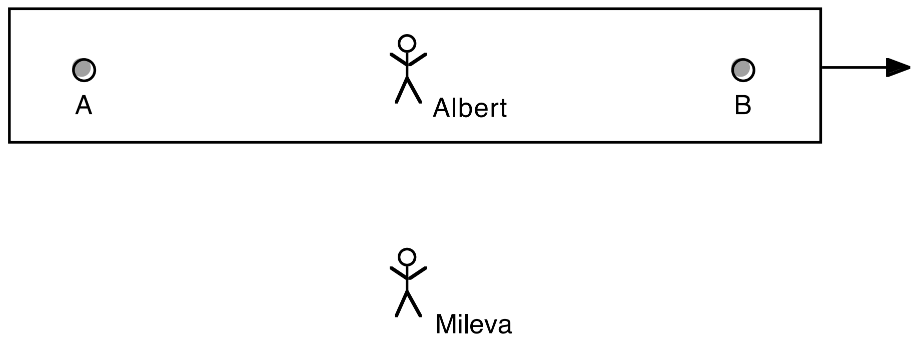
It is a little more complicated if we are not right in the middle of the two lights, but closer to light A, as shown in the above diagram. Because A is a lot closer, if A and B do flash at the same time, the light from A will reach our eyes before the light from B. We would not directly observe the lights flashing at the same time. However, since we know the speed of light, we could calculate how much time should be between the flashes if we know how much farther away B is than A. If that time delay matches what we observe, we can conclude that the lights did indeed flash at the same time.

The reference frame of the lights is anywhere that is not moving with respect to the lights. No matter where someone is in that reference frame, everyone will agree whether the two lights flash at the same time or not. If the lights flash at the same time, then people closer to A will "see" the flash from A first and people closer to B will "see" the flash from B first, but everyone will be able to do a little d = vt calculation and conclude that the lights did in fact have to start at the same time in order to arrive at their position with whatever time delay they experience.

Only the people who happen to sitting an equal distance from each light will actually "see" the lights flash at the same time; everybody else will have to do some calculations to conclude that the lights *must have flashed at the same time*. It turns out, if we are careful in how think about our reference frames, we can conceptually think about what happens without having to do any math.

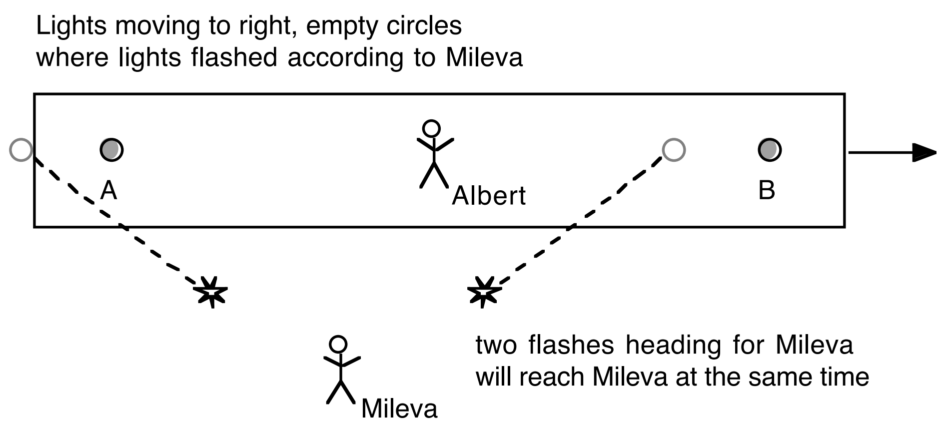
Flashing Lights in Different Reference Frames
Now let's begin the fun! We will step through a few situations involving multiple reference frames and observers of flashing lights. Let us always choose a convenient position in the reference frame to decide what is happening – let us try and put ourselves always equidistant from the two lights.

In the series of pictures and situations that follow, we will examine what two different reference frames "see" when looking at two lights flashing. Albert will always be in the reference frame of the lights, and situated in the special location for that reference frame exactly in the middle of the two lights, marked A and B. Mileva will be in the second reference frame. We will start off with Albert moving to the right, with respect to Mileva, as shown below.



Mileva feels that she is at rest, and that Albert and the lights are moving to the right. Albert feels that he and the lights are at rest, and that Mileva is moving to the left.

1) Mileva sees the lights flash at the same time.
The picture below shows the two lights flashing at the same time in Mileva's reference frame. We will put Mileva in the special location in her reference frame equidistant to the two lights the instant they flash.



Notice that once the lights flash, Albert is still moving to the right. The diagram tries to show where the lights were when they flashed, and the straight line paths they take to get to Mileva. Both Mileva and Albert will agree that the two lights hit Mileva at the same time.

Now comes the tricky part: in what order does Albert think the lights flashed? The picture below shows the two lights flashing at the same time according to Mileva, but this time traces the two rays of light heading to Albert.

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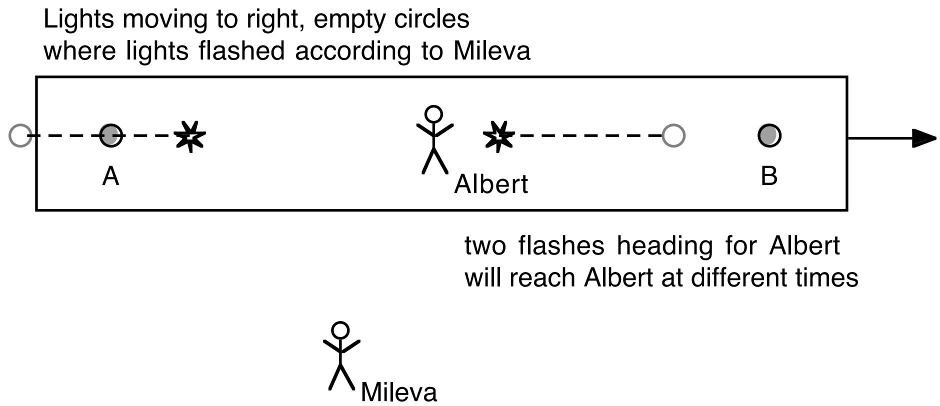
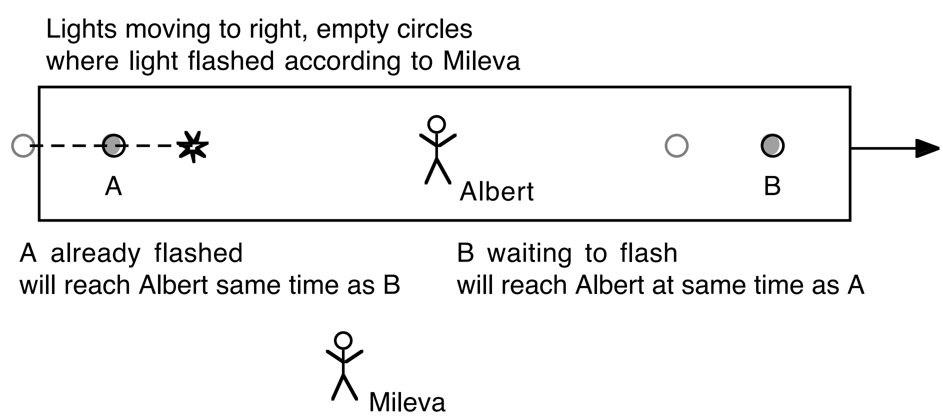


Because Albert and the lights are moving, once the lights flash, Mileva sees Albert running *away* from the flash from light A and *into* the flash from light B. Mileva sees the light from B hit Albert, then some time later, the light from A hit Albert. That means that Albert has to also see the light from B first, followed later by A. Since Albert is in the special location in his reference frame, he concludes that B flashed first, then A.

> Let's take the time to restate that: In Mileva's frame of reference, the two lights flashed at the same time, while in Albert's they flashed at different times. This turns out to be fundamental to relativity. **Events at different locations can only be simultaneous in one reference frame.** If Mileva sees the light flashes as simultaneous, then Albert has to see them as occurring at different times. What was simultaneous in one reference is not simultaneous in another!

2) Albert sees the lights flash at the same time.
So what happens if Albert sees the lights flash at the same time? In order for Albert to observe the lights flash at the same time, then photons from each light have to reach him at the exact same time. The picture below shows what Mileva would have to see in order for this to happen.



Because Mileva sees Albert moving to the right, in order for photons from A and B to hit Albert at the same time, A will have to flash first because Albert is traveling away from it. Only when the photon from A gets close to Albert will Mileva see B flash. Then she would see both the photons hit Albert at the same time.

In Albert's reference frame the two flashes were simultaneous, while in Mileva's A flashed first.

Another way to look at the situation is the following diagram from Albert's frame of reference.

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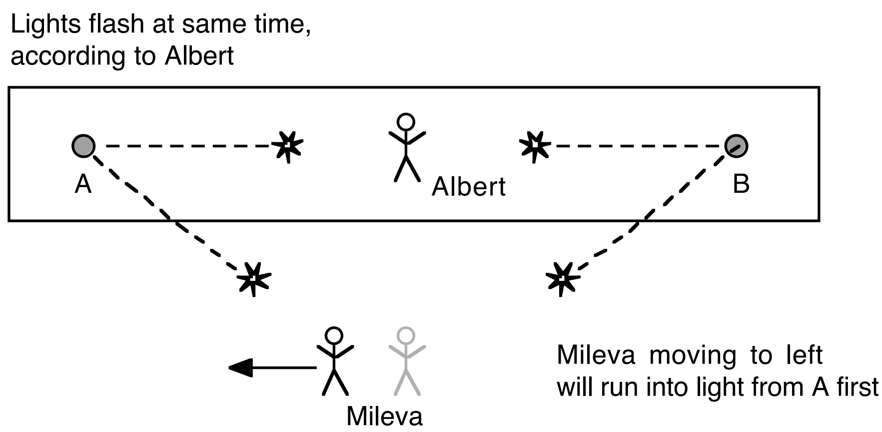

In Albert's reference frame, the lights flash at the same time and he is at rest. Albert sees Mileva moving to the left at high speed. The diagram tries to show where Mileva was when the lights flashed, and shows Mileva initially in the "special" location at the midpoint of the lights in her reference frame so that we need only judge the order that the lights hit her. When the lights flash at the same time in his reference frame, then he will see Mileva moving to the right, and so he will see her run into the flash from A first, and some time later, the flash from B will finally catch up to her. (Notice the similarity to when the lights flashed at the same time in Mileva's reference frame.)