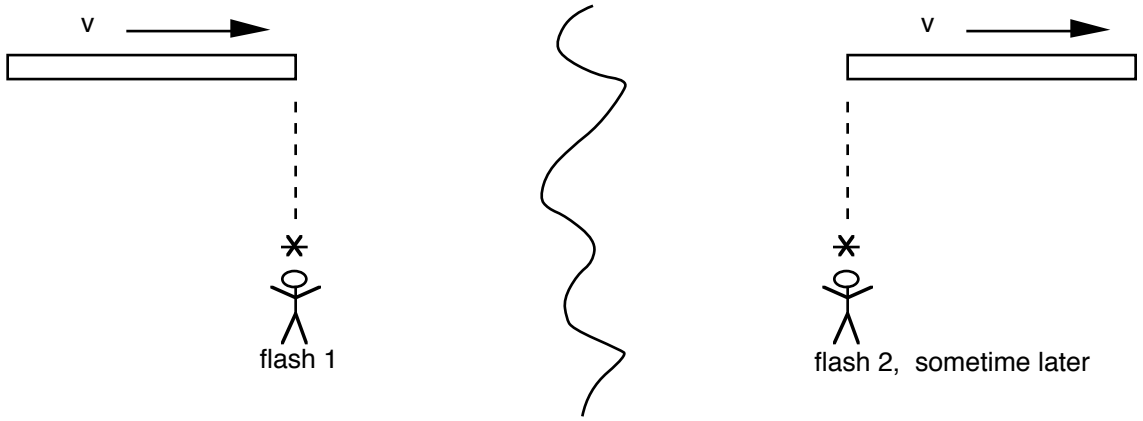
Length Contraction

Another surprising relativistic effect is the apparent shortening of objects that are traveling by at a speed close to that of light. It is easy to derive an expression for this contraction by thinking about an object traveling past a stationary observer. The observer will measure the length of the object by flashing a light when the front of the object goes by, then flashing the light again when the back of the object goes by. By measuring the time between the flashes, the observer can calculate the length of the object



The length of the object in its own reference frame we will call L_0. The length of the object as calculated by the observer we will call L. The time between the two flashes of light (which is simply the time it took for the length of the object travel past the observer) we will call t_0. Therefor the length of the object as calculated by the observer is

$$L = vt_0$$

From the reference frame of the object, it is at rest and the observer is traveling past it to the left with a speed of v. From the object's point of view, it has length L_0, and there was a flash of light at its front end, and then sometime later at its back end. However, the time between the flashes of light is not the same as that measured by the observer. The object would see the observer in slow motion, and therefor think that the time between the flashes was longer. The amount longer is the time dilation expression already found. From the object's point of view, its length is therefor

$$L_0 = vt \text{ where } t = \gamma t_0$$

Combining these expressions, we get

$$L = vt_0 = \frac{vt}{\gamma}$$

Which finally gives us the relationship for length contraction

Length Contraction:	$L = \frac{L_0}{\gamma}$

The observer sees the object shorter than when at rest! It is important to note that this contraction is only in the direction of travel. Also, remember that these relativistic effects are always "happening to the other person," in that each reference frame thinks the other is shortened in the direction of travel and has time running slow.