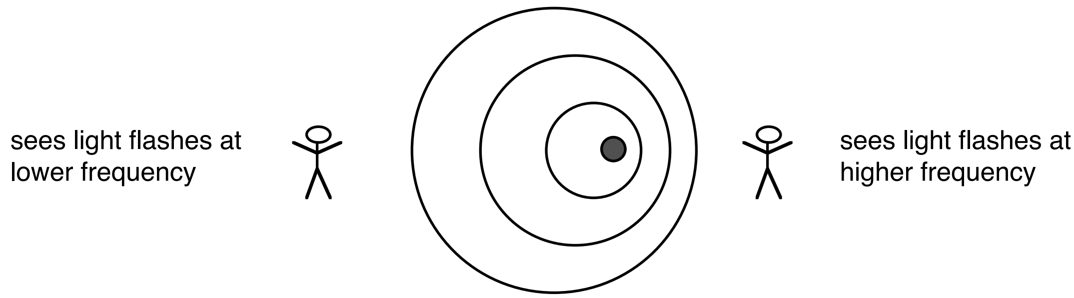
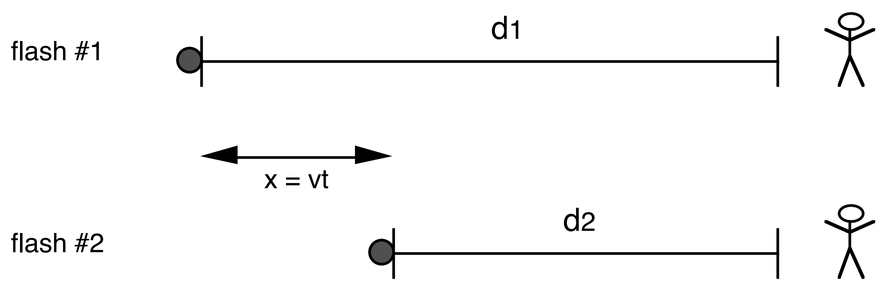
Doppler Shift

With sound, the Doppler Shift is the shift in frequency because a sound source is moving. If the sound is moving towards someone listening, then the person listening will hear the sound at a higher frequency, and if it is moving away, then it will be a lower frequency. (It works the same way if the listener is moving and the sound source is still.)



There is also a Doppler Shift in relativity, though it takes a slightly different mathematical form. From the above diagram, imagine a light that flashes at a constant frequency. If the light is moving towards someone, the person on the right in the diagram, then it would appear to that person the light flashes are happening at a higher rate, because the light pulses are getting squished together. If the light is moving away from someone, then it would appear that the light flashes are happening at a lower rate. Each pulse has to travel a further distance than the pulse just before.

To derive an expression for the Doppler shift, lets look at the time it will take a pulse to reach an observer, and then look at the time it will take for the next pulse to reach the observer. Let's first do this for the light pulse going towards the observer with speed v.



The first flash has to travel a distance d_1, while the second flash will have to travel a smaller distance, d_2. The pulses travel at the speed of light, c. The time it will take each pulse to travel those distances are thus

$$t_1 = \frac{d_1}{c} \qquad \text{and} \qquad t_2 = \frac{d_2}{c}$$

The distance d_2 is less than d_1 because the light has moved to the right. Since the light is moving with speed v, the distance the light travels between pulses is

$$x = vt \qquad \text{and so} \qquad d_2 = d_1 - vt$$

The time t is the time between flashes, as measured by the observer. The light itself thinks the time between the flashes is its proper time, t_0, where $t=\gamma t_0$, according to the standard time dilation formula.

The observer sees the first light pulse at time t_1 and then the second at time t_2+t. (The second pulse flashes a time t after the first one, and then takes a time t_2 to reach the observer.) Therefor the observer sees the time between the pulses, T, to be

$$T = (t_2 + t) - t_1$$

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Substituting in the time dilation formula and the expressions for d_2, t_1 and t_2 above, we get

$$
\begin{aligned}
T &= \frac{d_2}{c} + t - \frac{d_1}{c} \\
&= \frac{(d_1 - vt)}{c} + t - \frac{d_1}{c} \\
&= \left(1 - \frac{v}{c}\right)t \\
T &= \left(1 - \frac{v}{c}\right)\gamma t_0
\end{aligned}
$$

Since frequency is the inverse of the period, we can write the following expression for the frequency seen by the observer, where f_0 is the proper frequency of the "moving" lamp.

$$f = \frac{1}{\gamma\left(1 - v/c\right)} f_0 = \frac{\sqrt{\left(1 - v^2/c^2\right)}}{\left(1 - v/c\right)} f_0 \qquad \text{(objects approaching)}$$

The above formula is for the case of the lamp moving towards the observer. We could do a similar analysis for the lamp being still, and the observer moving towards the lamp, and we would get the same result. (If we didn't, then we would run the risk of being able to define a preferred reference frame, which cannot be.)

For the case in which the two objects are moving apart, again we would do the same analysis. The only difference would be that the distance traveled by the second pulse would be longer, so the time longer. (The above derivation is the exact same, except that $d_2 = d_1 + vt$) The end result is

$$f = \frac{\sqrt{\left(1 - v^2/c^2\right)}}{\left(1 + v/c\right)} f_0 \qquad \text{(objects receding)}$$

Note that this is the same except for the bottom term. It turns out that the relativistic Doppler shift for object approaching each other is just the inverse of the effect if they are receding. This derivation would be the same for the classical Doppler shift, except that there would be no "gamma" term, and for the classic Doppler shift, the shifts for approaching and receding are not inverses of each other.