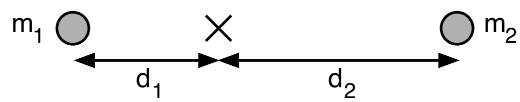
Center of Mass

Your book defines and shows the center of mass equations in the very beginning of chapter 9, but I would like to develop the ideas based off of our lab experience, and hopefully it will make a little more conceptual sense.

Imagine two different masses separated by some distance. Somewhere in between is their "center of mass." Let's pragmatically define it as the point where they would be balanced - like we did with the introduction to torques lab. The diagram below shows two masses - and the "x" in the middle is the center of mass.

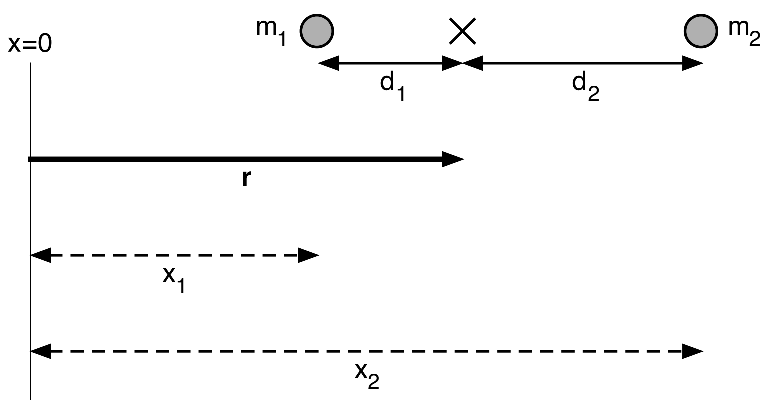


If the "x" is the center of mass, then the following would be true:

$$m_1 d_1 = m_2 d_2$$

That is the most basic way to think about the center of mass. It's just the weighted center of the masses. If you suspend an object from its center of mass, it will be balanced. (In a uniform gravitational field the center of mass is also the center of gravity.)

To show how this is the same as the book definition, let's just draw the same thing, but shifted along the horizontal axis. In the diagram below, **r** is the vector that points from the origin to the center of mass, x_1 and x_2 are the distances to the two masses.



Looking at the diagram, let's replace the d_1 with **r** - x_1 and d_2 with x_2 - **r**. Then we get

$$m_1(r - x_1) = m_2(x_2 - r)$$

Which we rearrange to get

$$(m_1 + m_2)r = m_2 x_2 + m_1 x_1$$

$$r = \frac{m_2 x_2 + m_1 x_1}{m_1 + m_2}$$

Your textbook just gives the above as the basic definition for the center of mass between two particles. This can then be generalized for lots of masses as (using M for the total mass)

$$r = \frac{\sum m_i x_i}{M}$$

We generalize that a little more into three dimension by simply doing the above three different times - one for each dimension. If we wanted to talk about an infinite number of infinitely small masses spread around, the above summation would be treated as an integral:

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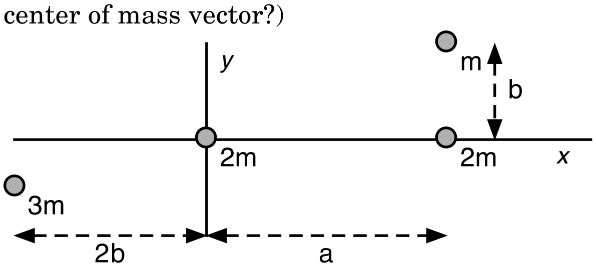
$$r = \frac{1}{M}\int x\,dm$$

In three dimensions we would just treat each dimension independently - so we would have three different summations (or integrals), one for each dimension.

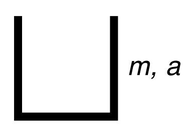
Why is the center of mass important? Basically, when we do Newton's Laws involving large objects, the Second Law applies to the center of mass. Add up all the external forces acting on an object as if they were acting on the center of mass. This is what we did earlier in the year - and it will still apply in the future when we look at rotation.

Questions:

1. Four point masses are arranged as shown in the diagram. Notice that the origin passes through one of the masses labeled "2m." Where is the center of mass? (Worded another way, what is the center of mass vector?)



2. Three identical, uniform masses each of mass m and length a are arranged as three sides of a square as shown in the diagram. Where is the center of mass?



3. Imagine that there are a bunch of different masses arranged on the x-axis, on either side of the origin. After doing $r = \frac{\sum m_i x_i}{M}$ for all the masses, you get r = 0. What does that mean?