Test: Gravity

Some equations you may need:

$$F = G\frac{m_1 m_2}{r^2} \quad \frac{T^2}{R^3} = \frac{4\pi^2}{GM} \quad v_e = \sqrt{\frac{2GM}{r}} \quad U = -\frac{Gm_1 m_2}{r} \quad E = -\frac{GmM}{2R}$$

$$F_c = \frac{mv^2}{r} \quad K = \frac{1}{2}mv^2 \quad L = I\omega \quad \vec{\tau} = \vec{r} \times \vec{F} \quad I = \sum mr^2 = \int r^2\, dm$$

Some constants you may need:

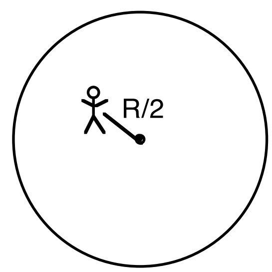
G = 6.67×10^{-11} Nm^2/kg^2 $M_{earth} = 6 \times 10^{24}$ kg $R_{earth} = 6.4 \times 10^6$ m $D_{earth\text{-}sun} = 1.5 \times 10^{11}$ m

$M_{sun} = 2 \times 10^{30}$ kg R_{sun} =

Multiple Choice: *Choose the letter of the best answer. 3 points each.*

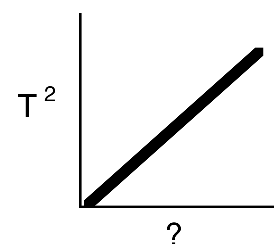
1. _____ If the sun were somehow smashed into a basketball, turning it into a black hole, what would happen to the earth?
   a. Tidal forces would be so much bigger that the earth would get ripped apart.
   b. The earth would get sucked into the black hole because of the increase in gravity
   c. Both a and b are correct.
   d. Not a whole heck of a lot, gravitationally speaking.

2. _____ Imagine you have a mass of m and are inside a hollow spherical shell of mass M and radius R. You are halfway between the center and the surface of the sphere, as shown. What is the net gravitational force on you?
   a. 4GmM/R^2. b. GmM/R^2. c. GmM/4R^2. d. GmM/2R^2.
   e. none of these are correct.



3. _____ An object at the surface of the earth weighs 90 N. Its weight at a distance 3R from the center of the earth is:
   a. 10 N. b. 30 N. b. 90 N. d. 270 N. e. 810 N.

Questions 4 and 5 refer to the following:
Some students are doing a lab in which they measured the orbital periods T and orbital radii R for some of the moons going around Saturn. After making a graph of T vs. R, they then linearized their results, creating the partial graph shown to the right.



4. _____ Which of the following would be the horizontal axis?
   a. R. b. R^2. c. 1/R^2. d. R^3.

5. _____ If the slope of the graph is equal to 1.04×10^{-15}, in standard SI units, what is the mass of Saturn?
   a. 9.6×10^{14} kg. b. 1.4×10^{25} kg. c. 5.7×10^{26} kg. d. 9.3×10^{29} kg.

6. _____ Neglecting air resistance, the escape speed from a certain planet of an empty space vehicle is 6000 m/s. What is the escape for the fully loaded space vehicle, which has triple the mass?
   a. 10,200 m/s. b. 2000 m/s. c. 18,000 m/s. d. 3530 m/s. e. 6000 m/s.

7. _____ Kepler's 2nd Law can be thought of as a restatement of which of the following
   a. The angular momentum of a planet orbiting the sun is constant.
   b. The gravitational force of the sun on a planet obeys an inverse-square relationship.
   c. Energy is conserved for an elliptical orbit.
   d. The area of a pie-shaped wedge can be approximated with a triangle.

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8. _____ An object is raised above the surface of the earth to a height of two earth radii above the surface of the earth. Then:
   a. its mass increases and its weight remains constant.
   b. both its mass and its weight remain constant.
   c. its mass remains constant and its weight decreases.
   d. both its mass and its weight decrease.
   e. its mass remains constant and its weight increases.

9. _____ Two masses have zero gravitational potential energy. Which of the following statements are true?
   I. The masses are infinitely far apart.
   II. The masses are touching each other.
   III. The masses can lose potential energy.
   a. I only. b. II only. c. III only. d. I & III only. e. II & III only.

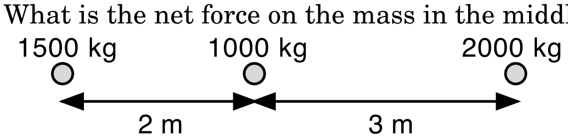
10. _____ A planet of mass m orbits a star of mass M. The distance between the planet and star varies from D to 2D. What is the total energy of the planet's orbit?
   a. $-\frac{GmM}{2D}$ b. $-\frac{GmM}{3D}$ c. $-\frac{2GmM}{3D}$ d. $-\frac{3GmM}{2D}$ e. $-\frac{GmM}{D}$

Bonus!
11. _____ If a friend where to complain to you that they had a headache because of "brain tides" and the fact that it was a full moon, you should
   a. commiserate with your friend, acknowledging how painful brain-tides can be.
   b. point out that brain-tides only depend on when the moon is overhead, and not on the actual phase of the moon.
   c. tell your friend to take a physics class.
   d. be rendered speechless by how stupid your friend is.
   e. realize I am very tired and there is no real answer to this question.

Problem Solving: *Show all work.*

12. The acceleration due to gravity on a random planet is 6.5 m/s^2. If the planet has a radius of 8,000 km, what is the escape velocity from the surface of the planet?

13. What is the net force on the mass in the middle of the diagram?



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14. Imagine there is a satellite orbiting the earth 3000 km above the surface of the earth.
    a. How many hours will it take to orbit the earth?

    b. If it was just dropped from that "height," how fast would it be going when it crashed into the earth? (Ignore air resistance, obviously.)

15. a. Show that the total energy for an object of mass m orbiting an object of mass M with an orbital radius of r is given by $E = -GmM/2r$.

    b. Using that result, what would be the slowest speed of a planet that took 4 years to go around the sun and had an orbital eccentricity of 0.3?

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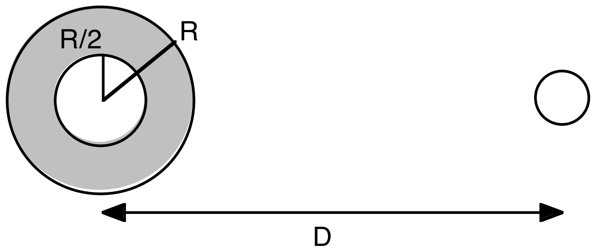
16. A sphere of mass M and radius R is a distance D away from a little mass m, as shown in the diagram. If the center of the big sphere were somehow hollowed out, so that there was an "empty" sphere of radius R/2 inside it, what would be the gravitational attraction between the two spheres? (The mass of the big sphere is M is before being hollowed out.)



17. Derive one of Kepler's Laws. (You can assume a circular orbit for Kepler's 3^{rd} Law, but not for the 2^{nd} Law.) Please use a little English to explain what you are doing.