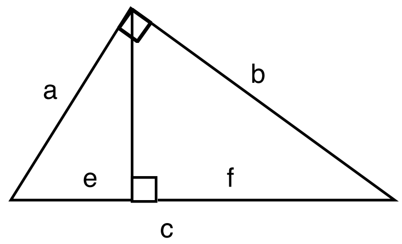
Pythagorean Theorem

There are many ways to prove the Pythagorean Theorem. Here are a few:

Triangle abc is a right triangle and an altitude is drawn, splitting side c into parts e and f.



By similar triangles, $\frac{e}{a} = \frac{a}{c}$ and $\frac{f}{b} = \frac{b}{c}$

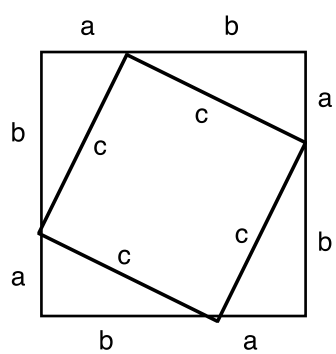
So that $a^2 = ec$ and $b^2 = fc$

$$a^2 + b^2 = ec + fc$$

Adding them together, we get $a^2 + b^2 = (e + f)c$

$$\therefore a^2 + b^2 = c^2$$

A square with side c is inscribed in another square, breaking the larger sides into a and b.



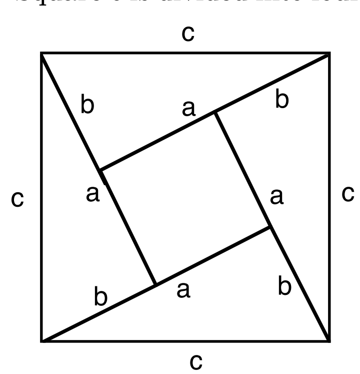
The area of the big square is equal to the area of the little square plus the area of the four right triangles, so that

$$(a + b)^2 = 4\left(\tfrac{1}{2}ab\right) + c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$\therefore a^2 + b^2 = c^2$$

Square c is divided into four right triangles (abc) and a small square (side = a-b).



The area of the big square is equal to the area of the little square plus the area of the four right triangles, so that

$$4\left(\tfrac{1}{2}ab\right) + (a - b)^2 = c^2$$

$$2ab + a^2 - 2ab + b^2 = c^2$$

$$\therefore a^2 + b^2 = c^2$$

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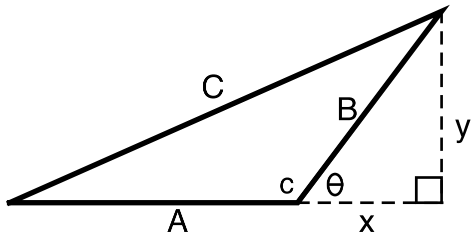
Pythagorean Theorem

Here are other useful extensions of basic trig stuff that can be useful in physics:

The Law of Cosines



We start of with a random triangle ABC as shown in bold to the left. Then we drop an altitude as shown by the dashed line y, making a right triangle with C as the hypoteneuse, and also a right triangle with B as the hypoteneuse.

We can use the Pythagorean Theorem on both right triangles:

$$C^2 = (A + x)^2 + y^2 = A^2 + x^2 + y^2 + 2Ax$$
$$B^2 = x^2 + y^2$$

We can then substitute the second into the first to get

$$C^2 = A^2 + B^2 + 2Ax$$

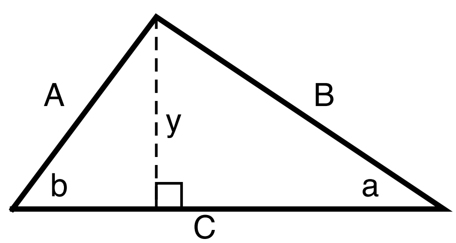
Now we notice that $x = B\cos\theta$ and $\theta = \pi - c$ so that we can say

$$x = -B\cos c$$

We finally substitute this into the expression earlier to get the Law of Cosines:

$$C^2 = A^2 + B^2 - 2AB\cos c$$

The Law of Sines



Given the triangle ABC shown above, we can drop an altitude to the base C with the dashed line y shown. Therefor we can say

$$y = A\sin b = B\sin a$$

Which we can rewrite as:

$$\frac{\sin a}{A} = \frac{\sin b}{B}$$

Continue this idea by rotating the triangle and using either A or B as the base, drop a new altitude, and repeat above, except we will be using C and the non base side A or B. So we end up with the Law of Sines:

$$\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}$$