Chapters 12, 13 & 14: **Universal Gravitation**

Text:

Chapter 12
Think and Explain:
Think and Solve:

Chapter 13
Think and Explain:
Think and Solve:

Chapter 13
Think and Explain:
Think and Solve:

Vocabulary:
mass, Newton's law of universal gravitation, gravitational constant, inverse square law, gravitational field, acceleration due to gravity, satellite, period, ellipse

Equations:

$$F_g = \frac{Gm_1 m_2}{d^2} \qquad v = \frac{2\pi r}{T} \qquad a_c = \frac{v^2}{r} \qquad F_c = \frac{mv^2}{r}$$

Constants: $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$ $m_{earth} = 6 \times 10^{24}\ kg$ $r_{earth} = 6.4 \times 10^6\ m$

Key Objectives:

Concepts
- ❑ What does it mean to say that gravity is universal?
- ❑ Why don't you feel the gravitational forces between you and other objects around you?
- ❑ Find by what factor the force increases or decreases based on increasing or decreasing the masses or distance.
- ❑ Apply Newton's 3^{rd} law to the concept of the forces between two masses.
- ❑ Relate the motion of a falling apple to the motion of the moon in orbit around the earth.
- ❑ Understand the concept of the gravitational field and how it relates to the acceleration due to gravity.
- ❑ Are objects in orbit around the earth truly weightless?
- ❑ [What causes ocean tides?]
- ❑ [What is a black hole?]

Problem Solving
- ❑ Calculate the force between two masses using Newton's law of universal gravitation.
- ❑ Solve for the missing variable when given all but one value.
- ❑ When given three masses in a row, find the net force on one of the masses due to the other two.
- ❑ Use universal gravitation to find the acceleration due to gravity on any planet when given the mass of the planet and the distance from the center of the planet.
- ❑ Apply the concepts of universal gravitation and circular motion to orbiting bodies.

Lab 15-1: Universal Gravitation

Purpose:
1. To gain a conceptual feel for how mass and distance affect gravitation.
2. To make and interpret graphs of Force vs Mass and Force vs Distance.

Procedure:

Part 1: Getting familiar with the software.

1. Start the simulation software "Gravity Force" by doing what your teacher told you to do. If you wish to avoid sarcastic comments, check the board before asking your teacher for help.
2. The simulation allows one to measure the gravitational force between two masses. Experiment with the program to see what you can control. What three things are you able to change?
3. Why are there so many zeros in the forces?
4. How can you measure the distance between the masses?
5. How can you reset everything to the original settings?

Part 2: Qualitative investigation.

6. What happens to the forces if you make m_1 bigger? How about smaller?
7. What happens to the forces if you make m_2 bigger? How about smaller?
8. What happens to the forces if you move the masses farther apart? How about closer?
9. Why are the two forces on the screen always equal and opposite?

Part 3: Quantitative investigations.

10. Reset everything to the original settings. Then change m_2 to 10 kg and record the force.
11. Fill out the rest of the data table by continuously adding 10 kg to the red mass and measuring the force. Make sure you don't change anything other than the red mass.
12. Repeat the above, but this time change m_1.
13. Make both masses 10 kg. Slide the blue mass to the left of the screen. Place the red mass 2 meters away from the blue one and record the force.
14. Fill out the rest of the data table by increasing the distance between the masses by 1 meter and recording the resulting force.

Data:

Mass 2 (kg)	*Force (N)*
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	

Mass 1 (kg)	*Force (N)*
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	

Distance (m)	*Force (N)*
2	
3	
4	
5	
6	
7	
8	
9	
10	

Lab 15-1: **Universal Gravitation**

Graphs: Make the following three graphs from your three sets of data: Force vs Mass 2, Force vs. Mass 1, Force vs. Distance.

Questions:

1. Compare the graphs you made with your answers from Part 2 on the other side. Do the graphs support what you thought was true?

2. How does the gravitaional force between the two masses depend on Mass 2?

3. How does the gravitaional force between the two masses depend on Mass 1?

4. Examine your data carefully - what happens to the gravitational force when you double the distance between the masses?

5. What happens if you triple the distance between the masses?

6. This is hard, but make an attempt at combining all those answers to come up with an expression that describes how the gravitational force between two masses depends on the actual masses and the distance separating them.

7. When you measure the distance between the masses, where exactly are you measuring to on the masses?

Calculators & Scientific Notation

It seems that the hardest part about the universal gravitation unit is student's struggles with scientific notation and their calculators. This sheet was designed to get you comfortable using your calculator. Please remember that every calculator is a little different, so we strongly suggest you remember to bring in your own scientific calculator for use - this way you know how to use it.

The key to successful scientific notation is using your calculator correctly. Every calculator has a method for typing in a number with scientific notation - but it is not always obvious. Look at your calculator - there will be one of the following buttons on it:

[EXP] [EE] [E] or [$x10^n$]

The first three buttons shown allow you to do scientific notation without using parentheses. You must use parentheses if your calculator has the fourth button. Again, your calculator will only have one of the shaded buttons - use whichever one your calculator has. The difficulties with scientific notation happen mostly when dividing numbers, and it is an issue of order of operations: you need to make sure the calculator "knows" exactly what to do. Try the following example

example $$\frac{2000}{4x10^2} = ?$$

If you have a "EXP", "EE" or "E" button on your calculator, press these buttons:

[2] [0] [0] [0] [÷]
[4] [EXP] [2] [=]

If you have the "$x10^n$" button, press these:

[2] [0] [0] [0] [÷]
[(] [4] [$x10^n$] [2] [)] [=]

Hopefully, you got 5 for an answer. If not, try again or ask your teacher for help. Now try these on your own to make sure you get the right answer. If you are still making mistakes, call your teacher over for a hand.

1. $\frac{3x10^4}{2x10^2}$

2. $\frac{5000}{7.5x10^3}$

3. $\frac{3x10^5}{(4x10^3)^2}$

4. $\frac{(5x10^9)(6x10^5)}{(4x10^9)^2}$

5. $(6.67x10^{-11})\frac{65(7.8x10^{23})}{(2.3x10^6)^2}$

Answers: 1) 150 2) 0.667 3) 0.019 4) 1.875×10^{-4} 5) 639.3

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Concept-Development Practice Page **13-2**

Force and Weight

1. An apple that has a mass of 0.1 kilogram has the same mass wherever it is. The amount of matter that makes up the apple

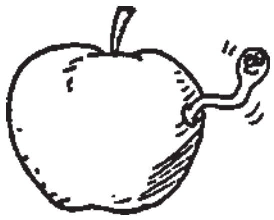
   (depends upon) (does not depend upon)

   the location of the apple. It has the same resistance to acceleration wherever it is—its inertia everywhere is

   (the same) (different).

   The weight of the apple is a different story. It may weigh exactly 1 N in San Francisco and slightly less in mile-high Denver, Colorado. On the surface of the moon the apple would weigh 1/6 N, and far out in outer space it may have almost no weight at all. The quantity that doesn't change with location is

   (mass) (weight),

   and the quantity that may change with location is its

   (mass) (weight).

   That's because

   (mass) (weight)

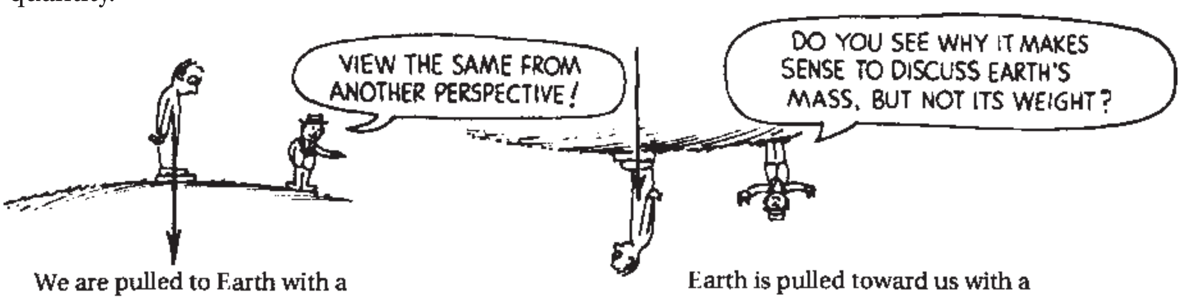
   is the force due to gravity on a body, and this force varies with distance. So weight is the force of gravity between two bodies, usually some small object in contact with Earth. When we refer to the

   (mass) (weight)

   of an object we are usually speaking of the gravitational force that attracts it to Earth.

   Fill in the blanks.

2. If we stand on a weighing scale and find that we are pulled toward Earth with a force of 500 N, then we weigh ________ N. Strictly speaking, we weigh ________ N relative to Earth. How much does Earth weigh? If we tip the scale upside down and repeat the weighing process, we can say that we and Earth are still pulled together with a force of ________ N, and therefore, relative to us, the whole 6,000,000,000,000,000,000,000,000-kg Earth weighs ________ N! Weight, unlike mass, is a relative quantity.



We are pulled to Earth with a force of 500 N, so we weigh 500 N.

Earth is pulled toward us with a force of 500 N, so it weighs 500 N.

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Concept-Development Practice Page **13-3**

Gravitational Interactions

The equation for the law of universal gravitation is

$$F = G\,\frac{m_1 m_2}{d^2}$$

where F is the attractive force between masses m_1 and m_2 separated by distance d. G is the universal gravitational constant (and relates G to the masses and distance as the constant π similarly relates the circumference of a circle to its diameter). By substituting changes in any of the variables into this equation, we can predict how the others change. For example, we can see how the force changes if we know how either or both of the masses change, or how the distance between their centers changes.

Suppose, for example, that one of the masses somehow is doubled. Then substituting $2m_1$ for m_1 in the equation gives

$$F_{new} = G\,\frac{2m_1 m_2}{d^2} = 2G\,\frac{m_1 m_2}{d^2} = 2F_{old}$$

So we see the force doubles also. Or suppose instead that the distance of separation is doubled. Then substituting $2d$ for d in the equation gives

$$F_{new} = G\,\frac{m_1 m_2}{(2d)^2} = G\,\frac{m_1 m_2}{4d^2} = \frac{1}{4}\,G\,\frac{m_1 m_2}{d^2} = \frac{1}{4}\,F_{old}$$

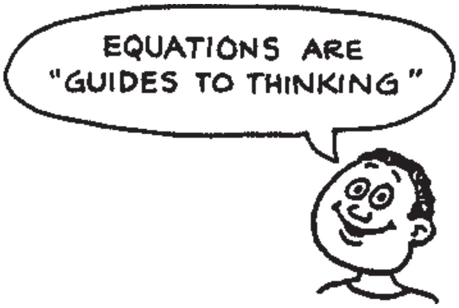
And we see the force is only 1/4 as much.

Use this method to solve the following problems. Write the equation and make the appropriate substitutions.

1. If both masses are doubled, what happens to the force?

2. If the masses are not changed, but the distance of separation is reduced to half the original distance, what happens to the force?

3. If the masses are not changed, but the distance of separation is reduced to one fourth the original distance, what happens to the force?



4. If both masses are doubled, and the distance of separation is doubled, show what happens to the force.

5. If one of the masses is doubled, the other remains unchanged, and the distance of separation is tripled, show what happens to the force.

6. Consider a pair of binary stars that pull on each other with a certain force. Would the force be larger or smaller if the mass of each star were three times as great when their distance apart is three times as far? Show what the new force will be compared to the first one.

Gravity, Part 1

Some useful numbers for this sheet. $G = 6.67 \times 10^{-11}\ N{\cdot}m^2/kg^2$

Mass of Earth: 6×10^{24} kg
Mass of Moon: 7.4×10^{22} kg

Radius of Earth: 6.4×10^{6} m
Radius of Moon: 1.74×10^{6} m

Distance Earth-Moon: 3.8×10^{8} m

Distance Earth-Sun: 1.5×10^{11} m

Conceptual Questions

1. What is Newton's Theory of Universal Gravitation? (Words and equation.)

2. Why does he call it universal?

3. What does the "d" represent in the equation?

4. What is "G?" (Words and number.)

5. If you get farther away from the earth, what happens to:
   a. the force of gravity acting on you?
   b. the acceleration due to gravity acting on you?
   c. your weight?

5. Why do we only notice the gravitational attraction to the earth, and not to the people and objects around us?

6. If you are standing on the earth, what is the distance between you and the earth, at least as far as universal gravitation is concerned?

Calculations

1. a. Calculate the gravitational force between the earth and the moon.

   b. Which experiences the greater force – the earth or the moon?

   c. Which experiences the greater acceleration – the earth or the moon?

   d. Why doesn't the moon crash into the earth?

Gravity, Part 1

2. a. What is gravitational force between two people (60 kg and 75 kg) who are 1 meter apart.

   b. Compared to other every day forces, how large is this attraction?

   c. Is it even noticable?

3. Calculate the gravitational force between you (m = 65 kg) and the earth.

4. If you were somehow floating in space 6.4×10^6 m above the surface of the earth, what would be the gravitational force on you? (Still use 65 kg)

5. There is a gravitational force of 100 N between two objects. What would be the gravitational force if
   a. the distance between them were doubled.

   b. the distance between them were halved.

   c. the distance between them were tripled.

   d. if the mass of one of the objects doubled.

   e. if the mass of both of the objects doubled.

   f. if the mass of one of the objects were halved.

   g. if the size of one of the objects were doubled (and the mass stayed the same.)

   h. if the size of both of the objects were doubled (and the mass stayed the same.)

Answers: 1. a) 2.05×10^{20} N b) the same c) the moon d) because it moves really fast in a nearly circular orbit; its acceleration is changing its direction of travel. 2. a) 3×10^{-7} N b) really, really small c) not a bit 3) 635 N 4) 159 N 5. a) 25 N b) 400 N c) 11.1 N d) 200 N e) 400 N f) 50 N g) 100 N h) 100 N

Gravity, Part 2

Some useful numbers for this sheet. $G = 6.67 \times 10^{-11}\ N{\cdot}m^2/kg^2$

Mass of Earth: 6×10^{24} kg
Mass of Moon: 7.4×10^{22} kg

Radius of Earth: 6.4×10^{6} m
Radius of Moon: 1.74×10^{6} m

Distance Earth-Moon: 3.8×10^{8} m

Distance Earth-Sun: 1.5×10^{11} m

1. The earth exerts a gravitational force of 7000 N on a satellite. What force does the satellite exert on the earth?

2. The Law of Universal Gravitation states that the gravitational force _____________ as the mass increases and ______________ as the distance increases.

3. Calculate the force of attraction between a 300 kg mass and a 550 kg mass that are 20 cm apart.

4. If the earth shrank to half its current size, but kept the same mass, how much would a 45 kg child weigh on the surface of the earth?

5. Imagine you and a friend are on a planet with a mass of 3.67×10^{24} kg and a radius of 7×10^{5} m.
   a. How much would you weigh on that planet on this planet if your mass were 60 kg?

   b. How much would your dog weigh on that planet if your dog's mass were 30 kg?

   c. How much would a big 120 kg rock weigh on that planet?

   d. Do you notice any pattern with your answers to parts a, b and c? (Hopefully you do.)

   e. What is the acceleration due to gravity on that planet?

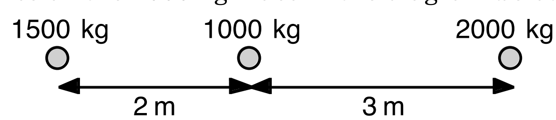
Gravity, Part 2

6. Two masses are placed so that their centers are 0.26 m apart. The force between them is 2.75×10^{-12} N.
   a. If one mass is 0.025 kg, what is the other mass?

   b. Calculate m_1 and m_2 if the masses are identical.

   c. Calculate m_1 and m_2 if $m_1=2m_2$.

7. Calculate the net force on the 1000 kg mass in the diagram below.



8. Instead of 2000 kg, what should the third mass be so that the net force on the middle mass is zero?

Answers: 1) 7000 N 2) increases; decreases 3) 0.000275 N 4) 1760 N
5. a) 300 N b) 150 N c) 600 N d) all 5x the mass; all proportional e) 5 m/s^2
6. a) 0.11 kg b) 0.053 kg c) 0.0373 kg & 0.0747 kg
7) 1.02×10^{-5} N to the left 9) 3375 kg

Family Guy Physics

We are going to attempt to learn some real physics by watching and analyzing a short clip from the TV show *Family Guy*. From the clip, we will calculate what must be Peter's mass if he did in fact have his own significant gravitational pull.

Watch the clip, and measure/estimate the following three pieces of information about the apple moving in an orbit around Peter.

Time for 5 Orbits: _________ Radius of Orbit: _________ Mass of Apple: _________

Calculations

1. What was the period of the orbit of the apple? (Put another way: how many seconds did it take the apple to go around Peter once?)

2. What was the linear speed of the apple? (Put another way: how fast was the apple moving around Peter?)

3. What was the centripetal acceleration of the apple as it traveled around Peter?

4. Calculate the centripetal force on the apple as it traveled around Peter?

5. Now comes the harder part. Where did this centripetal force come from?

6. So what was the gravitational force between the apple and Peter?

7. Finally, what would Peter's mass have to be in order for an apple to orbit around him as shown in the video?

8. In the clip, Brian also puts the TV in orbit around Peter. The mass of a TV is about 100 times the mass of an apple. Could the TV have stayed in the same orbit as the apple like it did in the video clip? (We will ignore the whole issue of how the TV was still on, because that is obviously not realistic.)

Answers: (Using 13 s, 1 m & 0.2 kg for the data) 1) 2.60 s 2) 2.42 m/s 3) 5.84 m/s^2
4) 1.17 N 5) gravitational attraction between apple and Peter 6) 1.17 N 7) 8.76 x 10^{10} kg

Orbits

Some useful numbers for this sheet.

Mass of Earth: 6×10^{24} kg
Mass of Moon: 7.4×10^{22} kg
Distance Earth-Moon: 3.8×10^{8} m

$G = 6.67 \times 10^{-11}$ $N \cdot m^2/kg^2$
Radius of Earth: 6.4×10^{6} m
Radius of Moon: 1.74×10^{6} m
Distance Earth-Sun: 1.5×10^{11} m

Concepts

1. Is an object in a perfect circular orbit with a constant speed accelerating? Explain.

2. Why is there a net force on a satellite that is orbiting the earth?

3. Why don't satellites simply fall and crash into the earth?

4. Using the terms "net force", "centripetal force" and "gravitational force" explain what has to happen for an object to be in a circular orbit.

Problems

1. Let's look at the orbit of the moon around the earth to go through the key ideas and steps in analyzing orbits.
   a. What is the force of gravity between the moon and the earth?

   b. Therefor, what is the centripetal force acting on the moon?

   c. How fast is the moon traveling around the earth?

   d. What is the period (in seconds) of the moon going around the earth?

   e. How many days it that?

Orbits

2. A 10,000 kg satellite is in orbit 8×10^6 m away from the center of the earth.
   a. What is the force of gravity between the satellite and earth?

   b. How fast is the satellite moving?

   c. How many hours will it take the satellite to go around the earth?

   d. If the satellite were 50,000 kg (5 times more massive) would your answers to *a*, *b*, and *c* change? Explain.

3. It turns out that you can figure out the mass of a planet by observing the motion of a moon going around the planet. Jupiter's moon Io goes around Jupiter in an orbit with a radius of 4.22×10^8 meters and a period of 1.77 days.
   a. How many seconds does it take Io to go around Jupiter?

   b. How fast is Io moving around Jupiter?

   c. What is the centripetal acceleration of Io around Jupiter?

   d. What is the mass of Jupiter?

Answers: 1. a) 2.05×10^{20} N b) 2.05×10^{20} N c) 1026 m/s d) 2.33×10^6 s e) 26.9 days
2. a) 62,560 N b) 7074 m/s c) 1.97 hours d) 5x the force of gravity, same speed, same period
3. a) 153,000 s b) 17,340 m/s c) 0.71 m/s^2 d) 1.89×10^{27} kg