Chapter 3-2: **Projectile Motion**

Text:
Chapter 3
Think and Explain: 1, 2, 6-10
Think and Solve: 1a, 2-6

Vocabulary:
component of velocity, vertical component of velocity, range, satellite

Equations:

$x = v_x t$

$y = v_{yi} t + 1/2\, g t^2$ *Note: These equations are really just* $d = vt$ *and* $d = v_i t + 1/2\, a t^2$*!*

$v_y = gt + v_{y_i}$

Constants: g = ±10 m/s^2

Key Objectives:
Concepts
- ❑ Identify the initial horizontal and vertical components of velocity for a projectile launched horizontally.
- ❑ State which velocity component changes over time and which component of velocity remains the same.
- ❑ Identify the velocity and acceleration at the highest point for a projectile launched at an angle on a level surface.
- ❑ Recognize and be able to sketch the motion graphs for a projectile. (x vs. t, y vs. t, v_x vs. t, v_y vs.t)
- ❑ State the launch angle that will yield maximum range.
- ❑ State the relationship between launch angles that will yield the same range.
- ❑ Given the paths of three projectiles, be able to describe the motion qualitatively and determine which projectile has the greatest time in air, horizontal velocity, initial vertical velocity, etc.
- ❑ Relate the motion of a satellite to the motion of a projectile.

Problem Solving
- ❑ Set up table and fill in given information for a horizontal projectile problem and solve for missing values.
- ❑ Set up table and fill in given information for a projectile launched at an angle problem and solve for missing values.
- ❑ For both type of projectile word problems (shot horizontally from an initial height, or shot from the ground) given the appropriate information you should be able to:
  - o find the time to the maximum height and/or the total time in the air.
  - o find the velocity components and speed initially, at its maximum height and when it lands.
  - o find the initial height and the maximum height of a projectile.
  - o find how far horizontally a projectile travels.
  - o calculate the speed of a projectile from the components of its velocity.
  - o calculate the components of a projectiles velocity from its speed and direction.

Lab 3-4: Projectile Motion off a Table

Purpose: Imagine a ball rolling across a table and then going over the side. It will land some distance away from the edge of the table because it was moving sideways. Obviously, it will travel farther if it is going faster, but does the time in the air depend on how fast the ball is is rolling across the table? That is what you must figure out.

Claim:

Evidence:

Reasoning:

Lab 3-5: Projectile Motion off a Table

Purpose:
1. To analyze the motion of a toy ball rolled off a table through video analysis.
2. To produce position and velocity graphs of a projectile's motion for horizontal and vertical components.

Procedure:
1. Make sure that the LabPro is NOT plugged into the computer. Open up Logger Pro. Under **Insert**, choose **Movie...**. Choose the correct movie. It will open up in the middle of the screen of Logger Pro.
2.  Enable video analysis by clicking on the box on the bottom right of the movie that looks like the button to the left.
3.  Set the scale of the movie by clicking on the "Set Scale" button (upper right corner), then clicking and dragging across the length of the horizontal black line, which is 0.5 meters long.
4.  Set the origin by clicking on the "Set Origin" button (upper right corner), and then clicking on the first position of the ball.
5.  Now to record the actual position of the toy ball for each frame of the movie, click on the "Add Point" button (upper right corner.) Carefully center the mouse on the toy ball, and click. Logger Pro will record the x and y coordinates of the mouse click, and the movie will automatically go the next frame. Do this for each frame of the movie. The ball will get smeared the faster it is moving - just click on the midpoint of the smear.
6. To clean up the window, under **Page**, choose **Auto Arrange**. You should now see the position vs. time graphs on the main screen.
7. To add the velocity vs. time graphs, under **Insert**, choose **Graph**. A floating window will appear with a new graph in it. Again, under **Page**, choose **Auto Arrange**.
8. To add the second velocity graph, click on the axis label (probably "Y Velocity" or "X Velocity") and then choose **More...** in the pop-up window that appears. Make sure both "X Velocity" and "Y Velocity" are checked off and then click **OK**.
9. You now have four graphs - one is a straight line with a positive slope and part of one is a straight line with a negative slope. Determine those slopes.
10. You will have to sketch the graphs and record the slopes in the questions below.

Questions:

First, let's go through the graphs for the horizontal motion, which are easier to interpret, but perhaps more surprising.

1. Sketch and interpret the horizontal position (x) verses time graph.

2. Sketch the graph of horizontal velocity(v_x) verses time. How do you interpret this graph, taking into account the graph of horizontal position verses time? Was there any acceleration in the x-direction?

3. Big Idea: Looking <u>only</u> at the graphs of x vs t or v_x vs t, could you tell whether the ball was rolling horizontally on the track or if it was falling?

Lab 3-5: Projectile Motion off a Table

The graphs of the vertical motion are harder to interpret, because they show two different things going on. Let's go through this:

4. Sketch the graph of vertical position (y) vs time here. Mark on the graph where the ball was rolling horizontally and where it was falling. Interpret what each part of the graph shows.

5. Sketch the graph of vertical velocity(v_y) vs time here. Mark on the graph where the ball was rolling horizontally and where it was falling. Interpret what each part of the graph shows.

Let's summarize these results:

6. While the ball was <u>on the track</u>:
   a. What was its horizontal velocity (v_x)?

   b. What was its vertical velocity (v_y)?

   c. What was its horizontal acceleration?

   d. What was its vertical acceleration?

7. While the ball was <u>in the air</u>:
   a. What was the horizontal velocity (v_x)? Did this change from when it was on the track?

   b. What was the initial vertical velocity (just as it left the table)? What happened to the vertical velocity (v_y) as it fell?

   c. What was its horizontal acceleration?

   d. What was its vertical acceleration?

Cliff Problems

1. A ball rolls off the edge of a table. It has an initial horizontal velocity of 3 m/s and is in the air for 0.75 seconds before hitting the floor.
   a. How high is the table?

   b. How far away (horizontally) from the edge of the table does the ball land?

   c. What are the horizontal and vertical components of the ball's velocity when it lands?

   d. How fast is the ball going when it lands?

2. The Coyote is chasing the Road Runner when the Road Runner suddenly stops at the edge of a convenient cliff. The Coyote, traveling with a speed of 15 m/s, does not stop and goes flying off the edge of the cliff, which is 100 meters high.
   a. How long is the Coyote in the air?

   b. Where does the Coyote land?

   c. What are the horizontal and vertical components of the Coyote's velocity when he lands?

   d. How fast is the Coyote going when he lands?

3. A car full of bad guys goes off the edge of a cliff. If the cliff was 75 meters high, and the car landed 60 meters away from the edge of the cliff, calculate the following:
   a. The total time the car was in the air.

   b. The initial velocity of the car. (Give the components.)

   c. The final velocity of the car just as it hits the ground. (Give the components.)

   d. The final speed of the car just as it hits the ground.

Answers: *1. a) 2.81 m* *b) 2.25 m* *c) v_x= 3 m/s & v_y = 7.5 m/s* *d) 8.1 m/s*
2. a) 4.47 s *b) 67.1 m* *c) v_x= 15 m/s & v_y = 44.7 m/s* *d) 47.2 m/s*
3. a) 3.87 s *b) v_x = 15.5 m/s & v_y = 0 m/s* *c) v_x = 15.5 m/s & v_y = 38.7 m/s* *d) 41.7 m/s*

More Cliff Problems

4. A ball is shot horizontally from a window. It has an initial horizontal velocity of 4 m/s and is in the air for 1.35 seconds before hitting the ground.
   a. How high is the window?

   b. How far away (horizontally) from the edge of the building does the ball land?

   c. What are the horizontal and vertical components of the ball's velocity when it lands?

   d. How fast is the ball going when it lands?

5. The Coyote is chasing the Road Runner when the Road Runner suddenly stops at the edge of a convenient cliff. The Coyote, traveling with a speed of 25 m/s, does not stop and goes flying off the edge of the cliff, which is 200 meters high.
   a. How long is the Coyote in the air?

   b. Where does the Coyote land?

   c. What are the horizontal and vertical components of the Coyote's velocity when he lands?

   d. How fast is the Coyote going when he lands?

6. A plane is flying across a level field and is 150 meters off the ground. When the plane is directly over point A, it releases a package, which then falls to the ground, and lands at point B, which is 500 meters away from point A. Calculate the following:
   a. The total time the package was in the air.

   b. The initial velocity of the package. (Give the components.)

   c. The final velocity of the package just as it hits the ground. (Give the components.)

   d. The final speed of the package just as it hits the ground.

Answers: *4. a) 9.1 m* *b) 5,4 m* *c) v_x = 4 m/s & v_y = 13.5 m/s* *d) 14.1 m/s*
5. a) 6.32 s *b) 158.1 m* *c) v_x = 25 m/s & v_y = 63.2 m/s* *d) 68 m/s*
6. a) 5.48 s *b) v_x = 91.3 m/s & v_y = 0 m/s* *c) v_x = 91.3 m/s & v_y = 54.8 m/s* *d) 106.5 m/s*

Projectile Motion Concept Sheet

Projectile motion is a combination of two separate motions: constant speed horizontally and constant acceleration due to gravity vertically. On this sheet, you will calculate what happens to the components of a projectile's velocity and position, and then graph the positions, much as you did on some previous concept sheets.

For this problem, we have a projectile launched upward with an initial horizontal velocity of 20 m/s and an initial vertical velocity of 30 m/s. Answer the following questions first:

1. What is the actual initial speed of the projectile?

2. What happens to the horizontal component of the velocity as the projectile flies through the air?

3. What happens to the vertical component of the projectile as it flies through the air?

4. At the projectile's maximum height, what is the horizontal component of its velocity?

5. At the projectile's maximum height, what is the vertical component of its velocity?

Now to fill out the chart on the other side by completing the following:

6. Fill out the column for the horizontal velocity (V_x) at each point in time. Explain how you filled the chart out, or show your calculations here.

7. Fill out the column for the vertical velocity (V_y) at each point in time. Explain how you filled the chart out, or show your calculations here.

8. Fill out the column for the horizontal position (X) at each point in time. Explain how you filled the chart out, or show your calculations here.

9. Fill out the column for the vertical position (Y) at each point in time. Explain how you filled the chart out, or show your calculations here.

NAME: ____________________

Projectile Challenge

Part I: Calculate how fast the launcher shoots projectile

Shoot launcher straight up and measure the maximum height of the ball.

Maximum height = ________ m

Use $v_f^2 = v_i^2 + 2\ a\ d$ where $a = -10\ m/s^2$

Vi = __________

Part II: Calculate how far away to place the cup.

Vi = Vx = __________

X = __________

Projectile Challenge

Part III: Optional Bonus (+1)

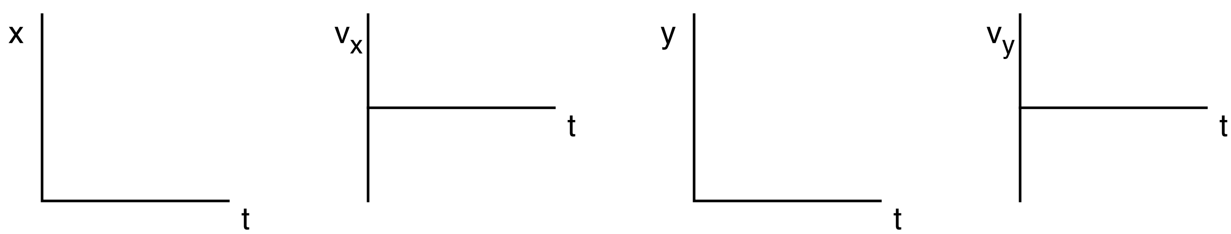
Combine the three equations $\bar{v} = \frac{d}{t}$ and $\bar{v} = \frac{v_f + v_i}{2}$ and $a = \frac{v_f - v_i}{t}$ to show that $v_f^{\,2} = v_i^{\,2} + 2ad$

Lab 3-6: **Projectile Motion**

Purpose:
1. To examine the motion of a projectile through the use of a camcorder.
2. To produce position and velocity graphs of a projectile's motion for horizontal and vertical components.
3. To analyze the motion of the projectile.

Procedure:
1. Two students will be video-taped tossing a tennis ball back and forth. This video will be converted into a small computer file, which will be analyzed using Logger Pro.
2. Make sure that the LabPro is NOT plugged into the computer. Open up Logger Pro. Under **Insert**, choose **Movie...**. Choose the correct movie. It will open up in the middle of the screen of Logger Pro.
3.  Enable video analysis by clicking on the box on the bottom right of the movie that looks like the button to the left.
4.  Set the scale of the movie by clicking on the "Set Scale" button (upper right corner), then clicking and dragging across the length of the meter stick on the wall.
5.  Set the origin by clicking on the "Set Origin" button (upper right corner), and then clicking on the first position of the tennis ball.
6.  Now to record the actual position of the tennis ball for each frame of the movie, click on the "Add Point" button (upper right corner.) Carefully center the mouse on the tennis ball, and click. Logger Pro will record the x and y coordinates of the mouse click, and the movie will automatically go the next frame. Do this for each frame of the movie.
7. To clean up the window, under **Page**, choose **Auto Arrange**. You should now see the position vs. time graphs on the main screen.
8. To add the velocity vs. time graphs, under **Insert**, choose **Graph**. A floating window will appear with a new graph in it. Again, under **Page**, choose **Auto Arrange**.
9. To add the second velocity graph, click on the axis label (probably "X Velocity" or "Y Velocity") and then choose **More...** in the pop-up window that appears. Make sure both "X Velocity" and "Y Velocity" are checked off and then click **OK**.
10. Sketch what the graphs look like in the space below. Make sure you label each graph. Three of the graphs should be lines; write down the slopes for those underneath the graph.
11. Answer the questions on the other side.

Graphs from Logger Pro



Lab 3-6: **Projectile Motion**

Questions:

1. The graph of horizontal position verses time is a straight line. What is the slope of the line, and what does the slope represent?

2. The graph of horizontal velocity verses time should be basically horizontal. How do you interpret this graph, taking into account the graph of horizontal position verses time? Was there any acceleration?

3. The graph of vertical position verses time is a curve – what does this graph tell you about the motion of the projectile? (It should look like a graph from an earlier lab, if that helps to interpret the graph.)

4. The graph of vertical velocity verses time is a straight line. What is the slope of the line, and what does the slope represent?

5. What were the components of the initial velocity of the ball?

6. What was the initial speed of the ball?

7. For an object that is caught at the same height from which it was thrown and ignoring air resistance
   a. what is true about the time needed to go up compared to the time needed to go down?
   b. what is true about the initial horizontal velocity compared to the final horizontal velocity?
   c. what is true about the initial vertical velocity compared to the final vertical velocity?
   d. what is its velocity at its maximum height? Be careful!
   e. what is its acceleration initially, at its maximum height and finally?

Projectile Motion Concept Sheet

Projectile motion is a combination of two separate motions: constant speed horizontally and constant acceleration due to gravity vertically. On this sheet, you will calculate what happens to the components of a projectile's velocity and position, and then graph the positions, much as you did on some previous concept sheets.

For this problem, we have a projectile launched upward with an initial horizontal velocity of 20 m/s and an initial vertical velocity of 30 m/s. Answer the following questions first:

1. What is the actual initial speed of the projectile?

2. What happens to the horizontal component of the velocity as the projectile flies through the air?

3. What happens to the vertical component of the projectile as it flies through the air?

4. At the projectile's maximum height, what is the horizontal component of its velocity?

5. At the projectile's maximum height, what is the vertical component of its velocity?

Now to fill out the chart on the other side by completing the following:

6. Fill out the column for the horizontal velocity (V_x) at each point in time. Explain how you filled the chart out, or show your calculations here.

7. Fill out the column for the vertical velocity (V_y) at each point in time. Explain how you filled the chart out, or show your calculations here.

8. Fill out the column for the horizontal position (X) at each point in time. Explain how you filled the chart out, or show your calculations here.

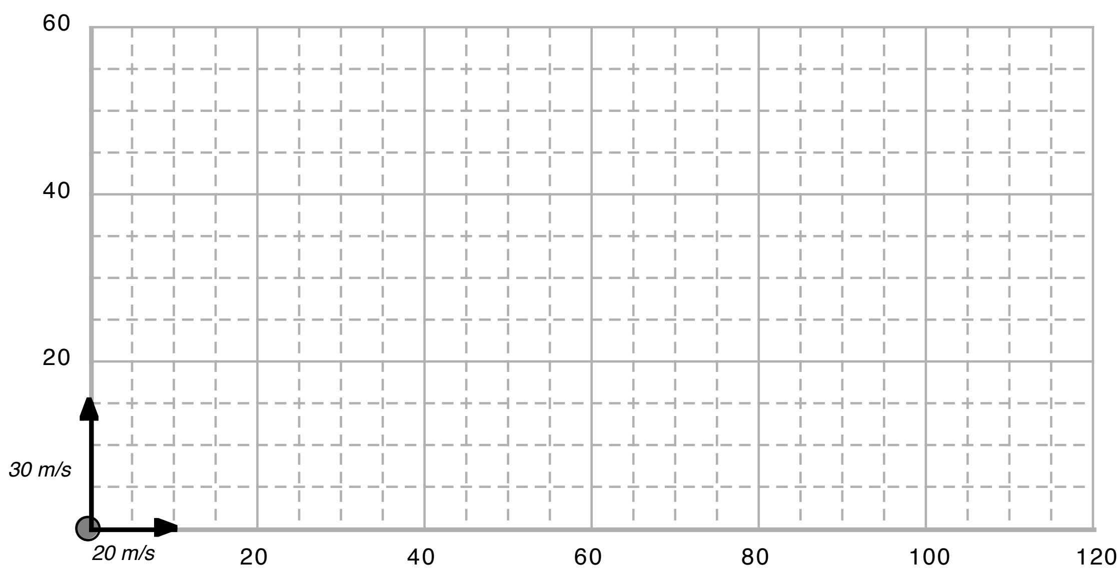
9. Fill out the column for the vertical position (Y) at each point in time. Explain how you filled the chart out, or show your calculations here.

Projectile Motion Concept Sheet

	Velocity		*Position*	
Time (s)	V_x *(m/s)*	V_y *(m/s)*	*X (m)*	*Y (m)*
0	20	30	0	0
1				
2				
3				
4				
5				
6				

10. Mark each of the positions of the projectile (X,Y) on the coordinate shown below. Label each position "t=" with the appropriate time. The first position is already done for you.

11. At each position, draw vectors to represent both components of the velocity. Use the scale of 1 square = 10 m/s. The first position is already done for you.



Projectile Motion Concept Sheet

Questions:

1. Imagine that you did the same thing for a projectile with an initial V_x of 10 m/s and V_y of 30 m/s.
   a. What would be different?

   b. What would be the same?

   c. How long would the projectile be in the air?

   d. What would be the maximum height of this projectile?

   e. How far away would the projectile land?

2. Imagine that you did the same thing for a projectile with an initial V_x of 30 m/s and V_y of 30 m/s.
   a. What would be different?

   b. What would be the same?

   c. How long would the projectile be in the air?

   d. What would be the maximum height of this projectile?

   e. How far away would the projectile land?

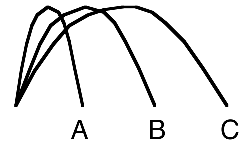
3. If you wanted the projectile to go higher,
   a. what should you change? Explain.

   b. would this affect the time in the air? Explain.

   c. would this affect how far away the projectile landed? Explain.

Projectile Motion Concept Sheet

4. Imagine that three different projectiles were launched across a level field. All the projectiles had the exact same maximum height, but they landed in different places. The paths of the projectiles are shown in the diagram to the right.

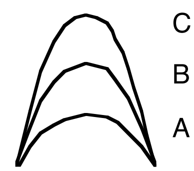


   a. Which projectile was in the air the longest time?

   b. Which projectile had the largest initial vertical velocity?

   c. Which projectile had the largest horizontal velocity?

5. Imagine that three different projectiles were launched across a level field. All the projectiles landed in the same place, but had different maximum heights. The paths of the projectiles are shown in the diagram to the right.

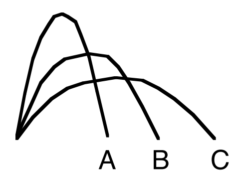


   a. Which projectile was in the air the longest time?

   b. Which projectile had the largest initial vertical velocity?

   c. Which projectile had the largest horizontal velocity?

6. Imagine that three different projectiles were launched across a level field. The projectiles all had different maximum heights and landed in different places. The paths of the projectiles are shown in the diagram to the right.

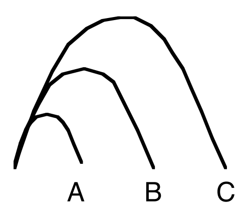


   a. Which projectile was in the air the longest time?

   b. Which projectile had the largest initial vertical velocity?

   c. Which projectile had the largest horizontal velocity?

7. Imagine that three different projectiles were launched across a level field. The projectiles all had different maximum heights and landed in different places. The paths of the projectiles are shown in the diagram to the right.



   a. Which projectile was in the air the longest time?

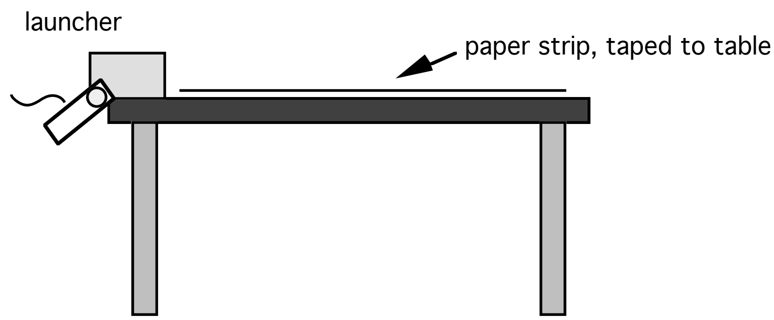
   b. Which projectile had the largest initial vertical velocity?

   c. Which projectile had the largest horizontal velocity? *(Be careful!)*

Lab 3-7: **Projectile Range**

Purpose:
1. To experimentally determine the initial launch angle that will give the maximum range of a projectile with a given initial speed.
2. To experimentally determine the relationship between angles that give the same range of a projectile with a given initial speed.
3. To use your experimental results to predict the landing position of a projectile for a given angle and to predict the angle to get a given landing position.

Materials:
1 projectile launcher
1 meter stick
1 paper strip
1 c-clamp
1 carbon paper



Procedure:
1. Clamp the projectile launcher to the end of your lab bench so that it will launch the ball bearing down your lab bench from the level of the table top. (Use the guide on the side of the launcher to see the initial launch position.)
2. Tape a strip of paper to the lab table so that the ball bearing will land on it.
3. As best you can, fire the projectile and record the range for 5º intervals, from 80º to 10º. You can assume that the angle of 90º and 0º will have a range of 0 cm. Fire the projectile to see about where it lands, place the carbon paper at that spot, and relaunch the projectile to measure its range. Try 3 launches per angle. (Angles less than 15º can be hard to do.)
4. Measure the distances to the average landing spot for each angle, and record in the data table.
5. Make a graph of Range vs. Angle. Make sure axes are labeled and your graph has a title.

Data:

Launch Angle (º)	Range (cm)	Launch Angle (º)	Range (cm)	Launch Angle (º)	Range (cm)
90	0	55		25	
80		50		20	
75		45		15	
70		40		10	
65		35		0	0
60		30			

Answer questions on other side.

Lab 3-7: Projectile Range

Conclusions:

1. Based on your data and graph, what is the relationship for launch angles that will have the same range?
2. Which angle will give the maximum range?
3. Would your results (questions 1 and 2) have worked if the projectile were fired off a cliff? Explain.

Questions:

A projectile is fired from the ground with an initial speed of 12 m/s.

1. What initial angle would give the greatest range for the projectile?
2. What initial angle would give the least range for the projectile?
3. What initial angle would give the greatest time in the air for the projectile?
4. What initial angle would give the least time in the air for the projectile?
5. Imagine it was fired at an initial angle of 35° and it lands someplace. What other angle would let the projectile land in the same place?
6. From question 5, which angle would be in the air longer?
7. What initial angle would give the greatest maximum height for the projectile?
8. What initial angle would give the least maximum height for the projectile?
9. What is the maximum range for this projectile? (*Warning! Several steps: angle=? components=? t=? and finally x=?)*

Projectile Motion Problems

1. A student tosses an eraser to his friend. The initial horizontal velocity of the eraser was 4.5 m/s and the initial vertical velocity was 5.36 m/s. The friend catches the eraser at the same level from which it was tossed.
   a. How long was the eraser in the air?

   b. How far apart were the two friends?

   c. What was the maximum height of the eraser?

   d. What were the components of the velocity at the top of its flight?

2. A kangaroo is jumping across a field in the outback. The kangaroo jumps with an initial horizontal velocity of 8 m/s and an initial vertical velocity of 5 m/s.
   a. What was the initial speed of the kangaroo?

   b. How long was the kangaroo in the air?

   c. What was the maximum height of the kangaroo?

   d. What was the horizontal distance of the kangaroo's jump?

3. Mary throws a ball to Suzy, who is standing 25 meters away. Suzy catches the ball from the same height at which it was thrown. If the ball was in the air for 4 seconds, calculate the following:
   a. Horizontal velocity.

   b. Maximum height of the ball.

   c. Initial vertical velocity.

   d. What happens to the components of the velocity and the acceleration as the ball flies through the air?

Projectile Motion Problems

4. Larry tosses a volleyball to his wife, Lise, who catches it at the same height from which it was tossed. The volleyball has an initial velocity of 15 m/s at an angle of 30° above the horizontal.
   a. What are the components of the initial velocity?

   b. How many seconds does it take the volleyball to reach its maximum height?

   c. How far apart are Lise and Larry?

   d. What was the acceleration of the volleyball after 1 second? Give the magnitude and direction.

*5. An astronaut on the moon tosses a rock with an initial velocity of 3 m/s at an angle of 35° above the horizontal. The acceleration due to gravity on the moon is 1.7 m/s^2.
   a. What were the components of the initial velocity of the rock?

   b. How long was the rock "in the air?'

   c. What was the maximum height of the rock?

   d. What was the horizontal distance traveled by the rock?

Answers:

1. a) 1.07 s b) 4.82 m c) 1.44 m d) v_x = 4.5 m/s & v_y = 0 m/s

2. a) v = 9.43 m/s b) 1.0 s c) 1.25 m d) 8 m

3. a) 6.25 m/s b) 20 m c) 20 m/s up d) v_x = constant = 6.25 m/s & acceleration = constant = 10 m/s^2 down & v_y starts positive 20 m/s (up) decreases to 0 m/s at top and continues to decrease to -20 m/s (down) when finally caught

4. a) v_x = 13 m/s & v_y = 7.5 m/s b) 0.75 s c) 19.5 m d) acceleration = gravity = 10 m/s^2 down

5. a) v_x = 2.46 m/s & v_y = 1.72 m/s b) 2.02 s c) 0.87 m d) 4.97 m